Condensed Matter via General Relativity

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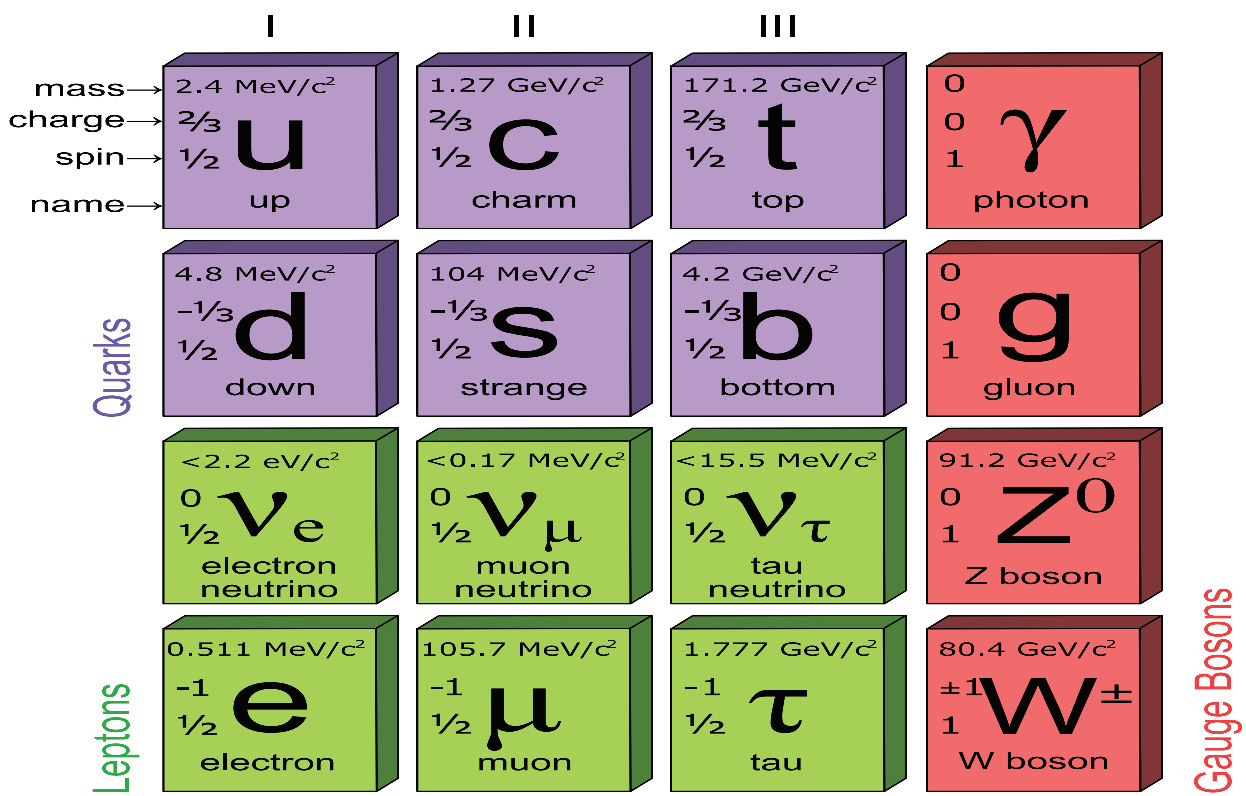
Key Ingredient

- **Gauge-gravity duality**

  - Yang-Mills gauge theory
  - General relativity as a theory of gravity

Gauge Theory

Standard Model of Particle Physics consists of SU(3) x SU(2) x U(1) Yang-Mills gauge theory



Overview

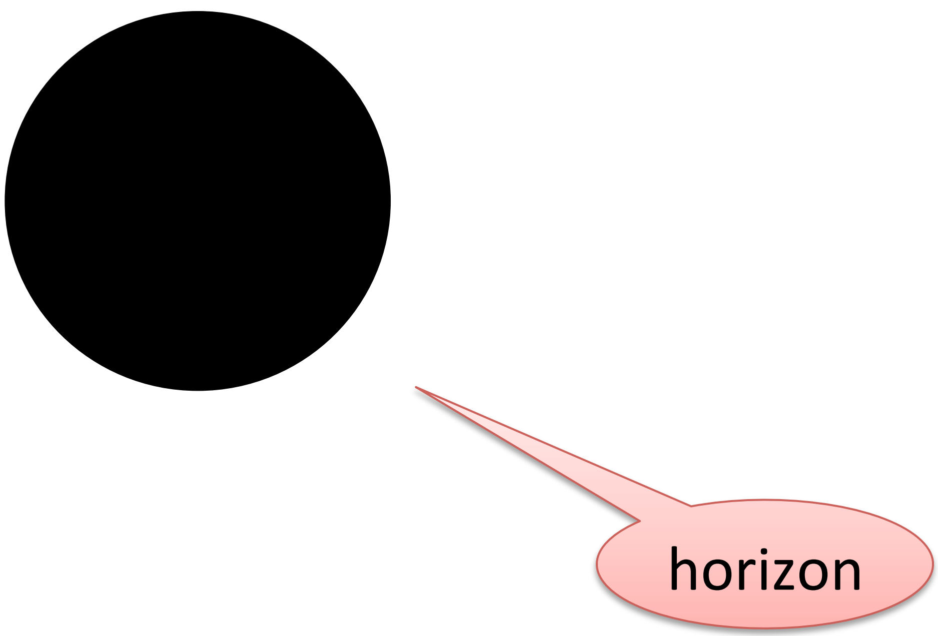
- Gauge-gravity duality
  - Strongly coupled quantum gauge theory
  - Classical general relativity

- Strongly coupled dynamics in CM physics, e.g.
  - High Tc superconducting materials
  - Strange, Bad, and, critical metals

- Aim is to identify **universal behavior**
  - Phase structure
  - Critical Behavior

Black Holes

- Solutions in general relativity (Schwarzschild)
- No physical object, once it has fallen past the horizon of a black hole, can escape again.

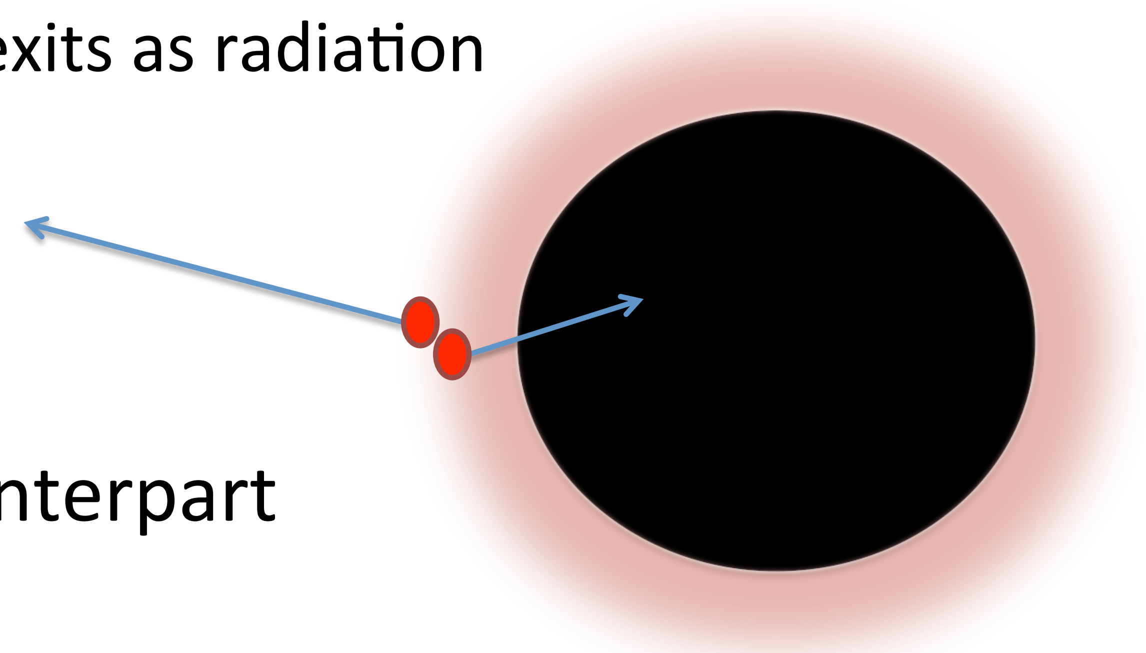


Hawking Radiation

- Quantum effects make black holes radiate.

  Virtual particle pairs can break up

  - One disappears inside BH
  - The other exits as radiation



- No classical counterpart

Black Hole Thermodynamics

- Black hole radiation spectrum is **thermal.**
- Bekenstein-Hawking relate

temperature T to acceleration κ at horizon

entropy S to area A of horizon

$$T = \frac{\hbar\kappa}{2\pi c} \qquad S = \frac{A}{4\ell_P^2} \qquad \ell_P = \left(\frac{\hbar G}{c^3}\right)^{\frac{1}{2}}$$

Area and Entropy

- Area (A) + Area (B) < Area (A+B) (Hawking)

- Entropy of astronomical black holes is huge

$$S_{\text{sun BH}} \sim 10^{18} \times S_{\text{sun}} \qquad \ell_P \sim 10^{-33}\text{cm}$$

- Micro-states ?

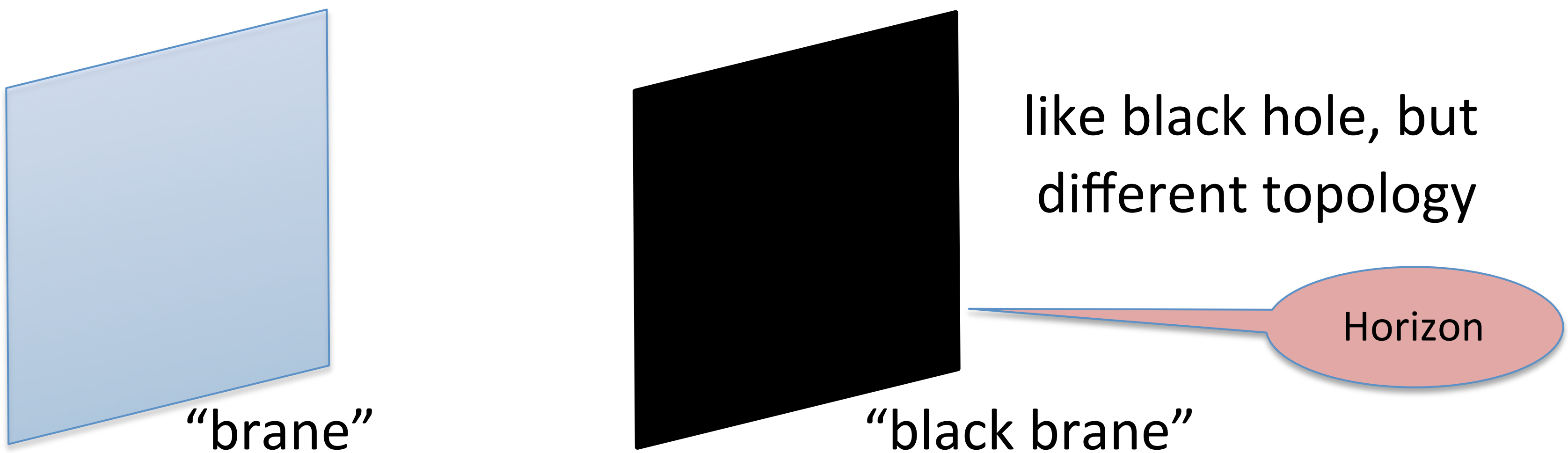
String Theory

- Extension from point particle to string produces drastic increase in the number of available micro-states,

- Automatically contains gravity (closed strings)
- Open strings produce Yang-Mills gauge fields
- Requires extra space-time dimensions, D=10
- Strings are tiny $\ell_s \approx 10^{-33}\mathrm{cm}$

Branes

- String theory also contains “branes”



- Generalize membranes;
  typically flat along brane, curved transverse space

Black Hole Statistical Mechanics

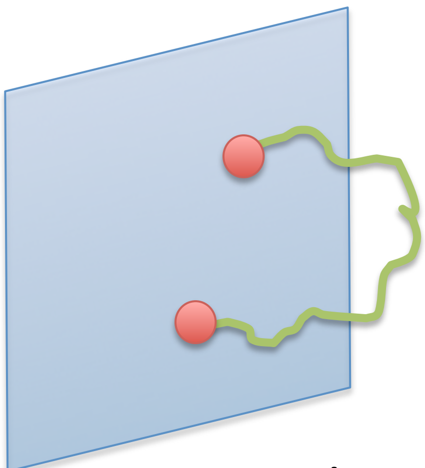
- Entropy of very special (supersymmetric) black holes, can be accounted for exactly by brane microstates (Strominger, Vafa)

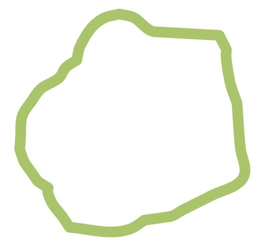
- String microstates numerous enough to account for entropy of arbitrary black holes. (Horowitz, Polchinski)

Onto gauge/gravity-duality …

D-Branes

- Theory of <u>closed strings</u>



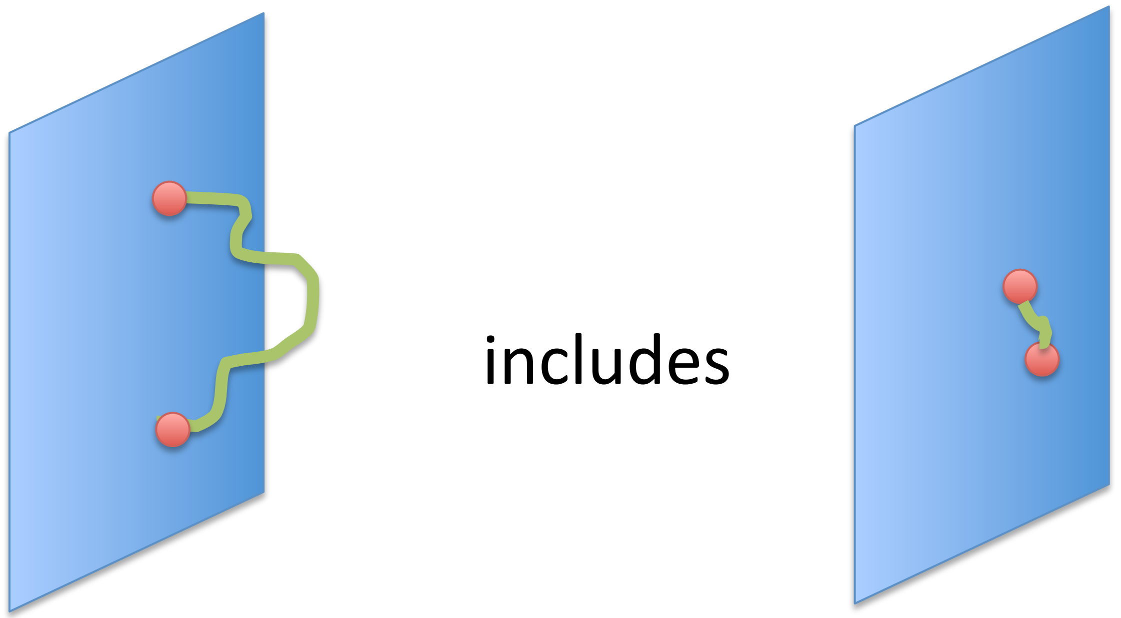


away from the D-brane:
only closed strings propagate

open string end points
are confined to lie on the D-brane,
(enforced by Dirichet boundary conditions)
but may move freely along the D-brane (Polchinski)

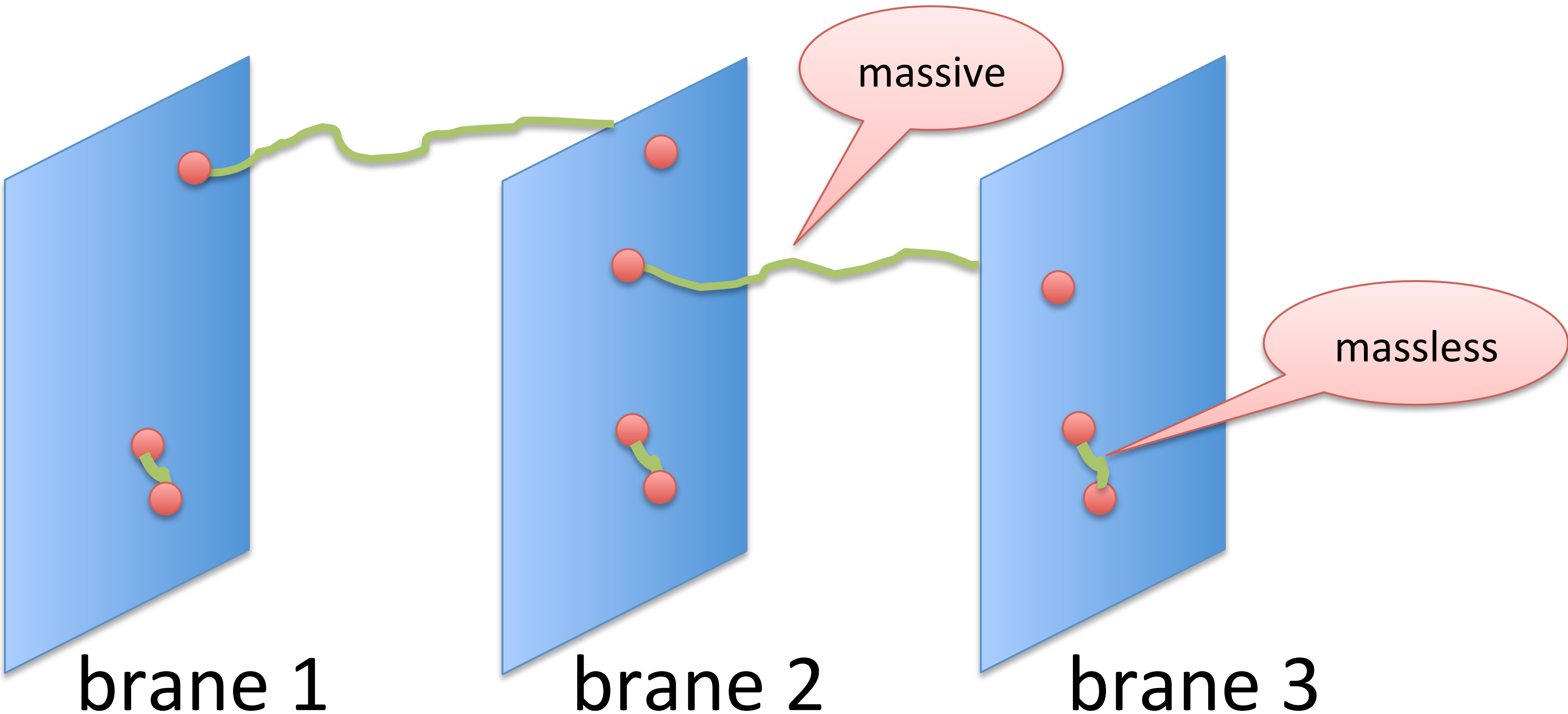
Branes carry Gauge Fields

- Mass of open string is proportional to length



➢ D-brane carries a (massless) Maxwell field, localized on the brane (not in the bulk)

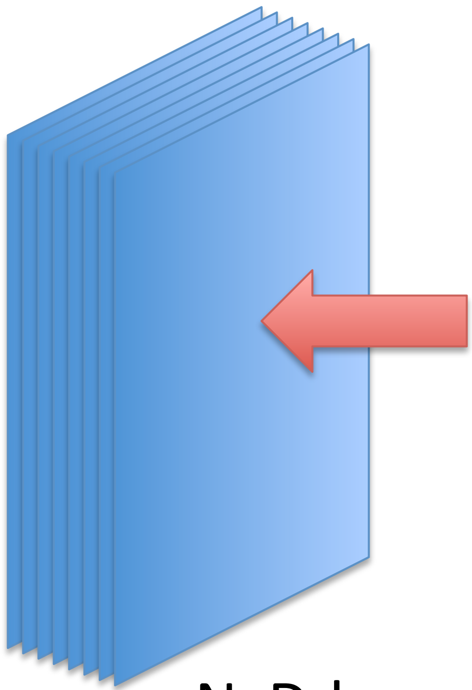
Multiple branes carry Yang-Mills



Strings -> Yang-Mills field

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$$

The Maldacena Limit



N D-branes
4-dim Minkowski
space-time world-volume

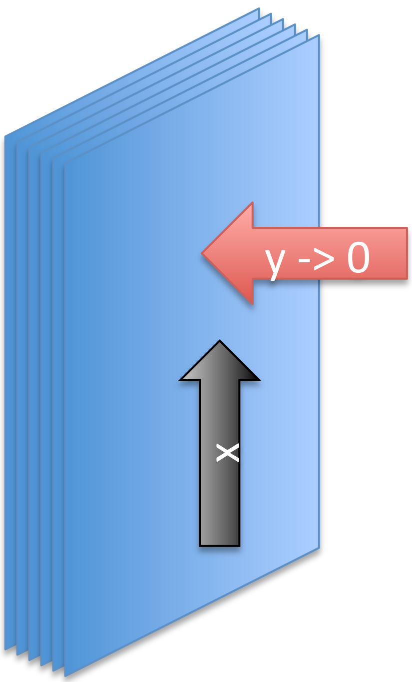
Zoom into geometry near brane, keep all physical parameters fixed, and let $\ell_s \to 0$

Resulting limit is equivalently given by:

* bulk gravity decouples: only U(N) gauge theory on branes remains
* string theory in the limit of near-horizon geometry

Near-horizon geometry

- Geometry of D-branes is given by their metric



$$ds^2 = f^{-1/2} dx^\mu dx_\mu + f^{1/2}(dy^2 + y^2 ds^2_{S^5})$$

$$f = 1 + \frac{L^4}{y^4} \qquad L^4 = g^2 N \ell_s^4$$

$$\lim_{\ell_s \to 0} \frac{ds^2}{\ell_s^2} = g^2 N ds^2_{AdS_5} + g^2 N ds^2_{S^5}$$

Near-horizon geometry is $AdS_5 \times S^5$

Maldacena Conjecture

Maximally supersymmetric U(N) gauge quantum field theory in 3+1-dim Minkowski space-time

is equivalent to

Quantum string theory on $AdS_5 \times S^5$

9. **The Large N limit of superconformal field theories and supergravity.**
Juan Martin Maldacena (Harvard U.). Nov 1997. 19 pp.
Published in **Adv.Theor.Math.Phys. 2 (1998) 231-252**
HUTP-98-A097
e-Print: **hep-th/9711200** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service; ATMP Server; Mathematical Reviews
Detailed record - Cited by 8788 records

Onto General Relativity

- String theory on $AdS_5 \times S^5$ very complicated
- Limits lead to weaker forms

Quantum strings ➛ Classical strings $N \to \infty$

Strings ➛ General Relativity $g^2 N \to \infty$

➢ Strongly coupled Quantum gauge theory

≡ Classical General relativity on $AdS_5 \times S^5$

Condensed Matter ?

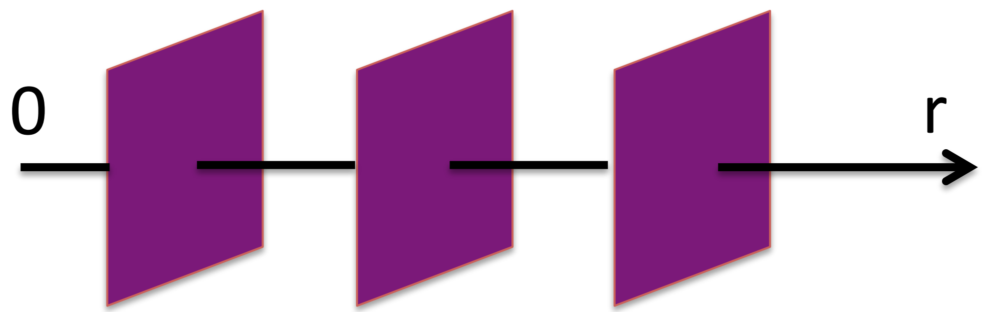
So far, this is not looking good:

- Poincaré invariant
- (maximally) Supersymmetric
- Zero temperature
- Zero charge density
- In fact: scale invariant

We will now remove these obstacles, and make gauge/gravity duality friendly to Condensed Matter.

AdS_5

$$ds^2_{AdS_5} = \frac{dr^2 - dt^2 + d\vec{x}^2}{r^2}$$



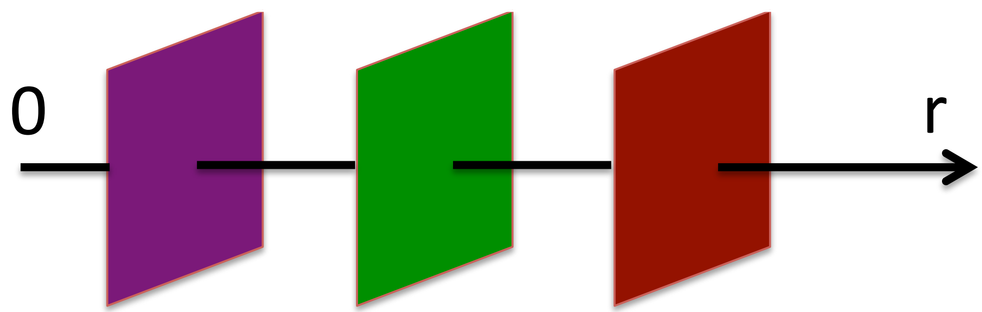
At fixed r, copy of flat Minkowski space-time

- Scale invariant metric (r, t, x) -> (br, bt, bx)
- Coordinate r = scale of system
- ➢ Scale invariant gauge theory dual

Renormalization group flow

- Let metric only asymptotic to AdS_5 as r-> 0

  Increasing r moves gauge theory from UV to IR



- Gauge-gravity gives geometrical realization of RG

➢ Gauge theory does not need to be scale invariant

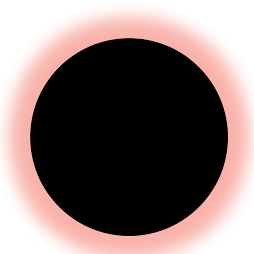
(Gubser, Klebanov, Polyakov; Witten)

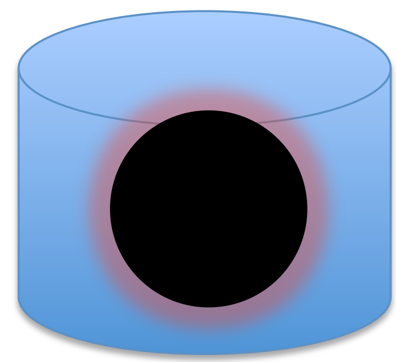
Finite Temperature

- "insert" a black hole/brane in AdS_5 (Witten)

$$T_{\text{gauge theory}} = T_{\text{Hawking}}$$

- AdS - bonus





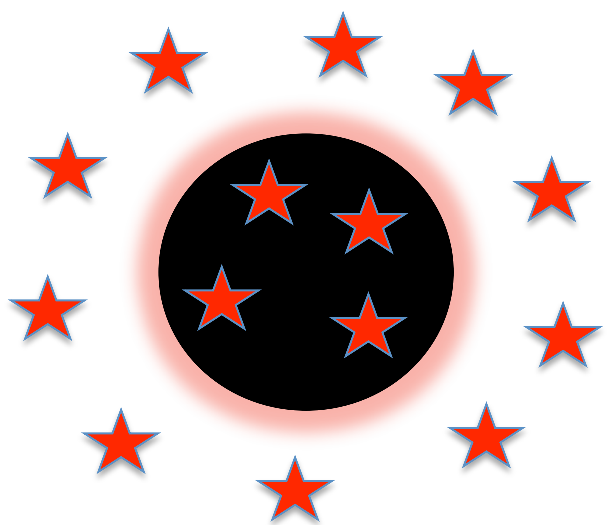
AdS_5

Schwarzschild black hole is NOT in equilibrium with radiation

AdS_5 black hole CAN BE in equilibrium with radiation

Charge density and magnetic field

- Include a Maxwell field in general relativity
  - Black hole can carry part of charge density



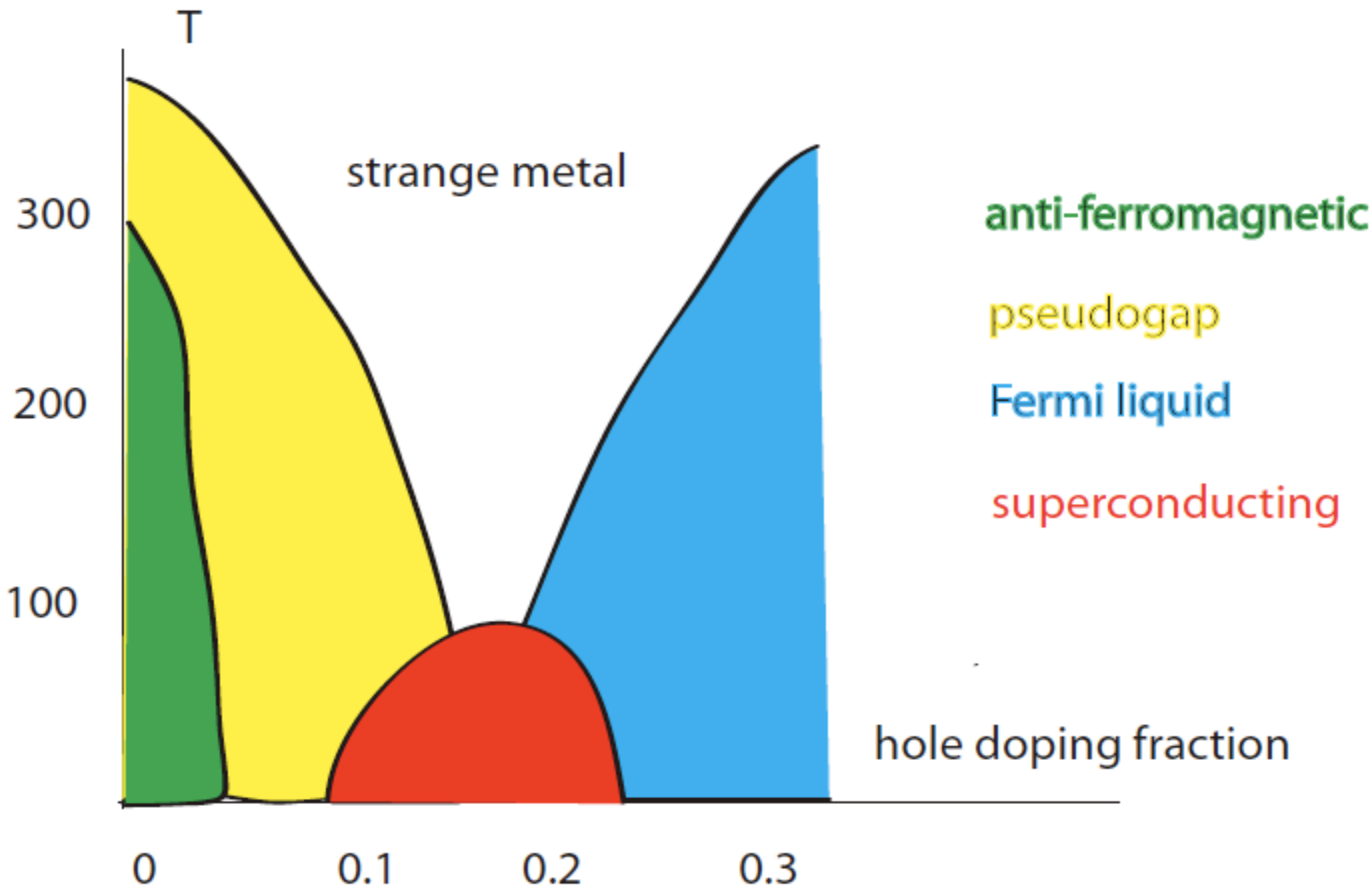
Applicability in condensed matter

- Ideally suited for
  - Low temperature, low frequency, low wavenumber
    (where numerical simulations slow down)
  - Processes in real (Minkowski) time
  - Transport, hydrodynamics, plasmas
    (early on by e.g. Policastro, Son, Starinets;
    reviews by Hartnoll, Hertzog, Sachdev, ...)
  - Quantum Criticality

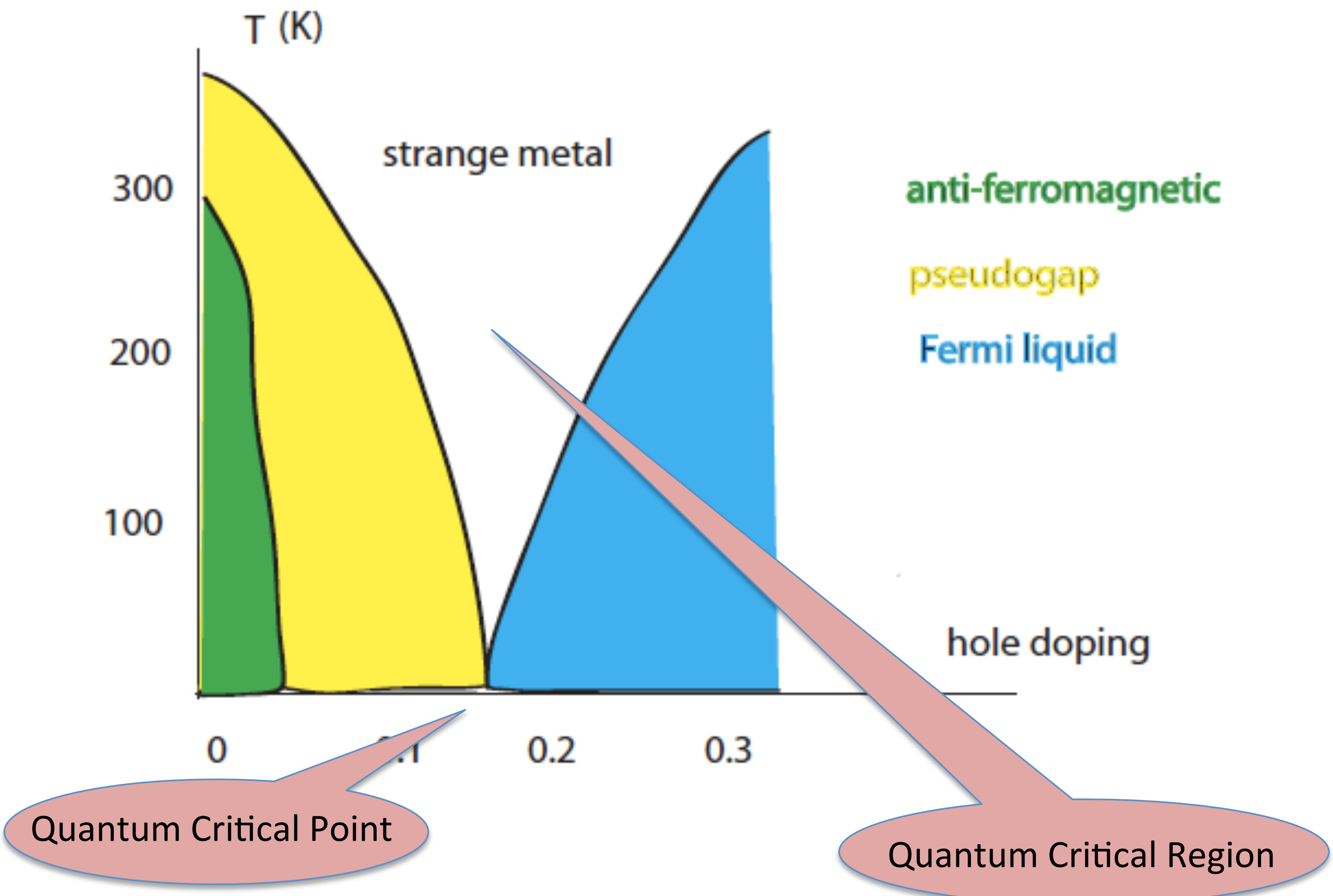
Quantum Criticality

- Quantum phase transition occurs as T=0
  - Macroscopic rearrangement of ground state
  - Driven by quantum fluctuations alone
- Quantum criticality T>0
  - Effects of quantum phase transition influence T>0
  - In quantum critical region, quantum and thermal fluctuations compete
- Suspected to be widely relevant at low T
  - E.g. in high T_c superconductors, like cuprates

High T_c superconductors



Conjectured quantum critical point



Fun with Magnetic Quantum Criticality

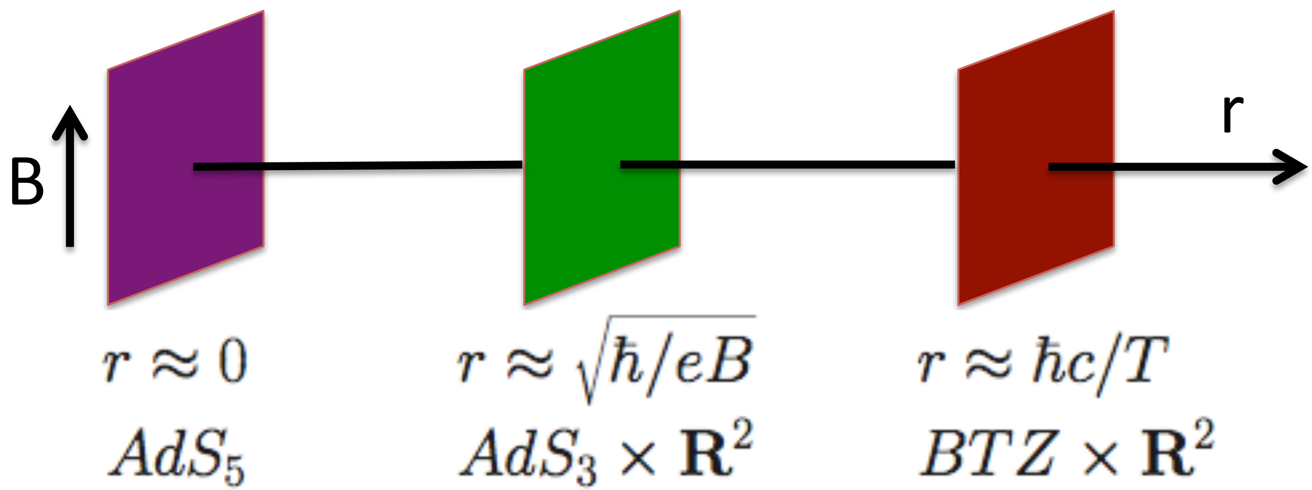
- Introduce universal physical parameters
  - Finite temperature T
  - Finite uniform charge density ρ
  - Uniform magnetic field B

- Gravity realization
  - Metric and Maxwell field
  - Chern-Simons coupling (natural from gravity side)
    - Charge of carriers proportional to B

B-field, no charge density

$B << T^2$ magnetic field irrelevant

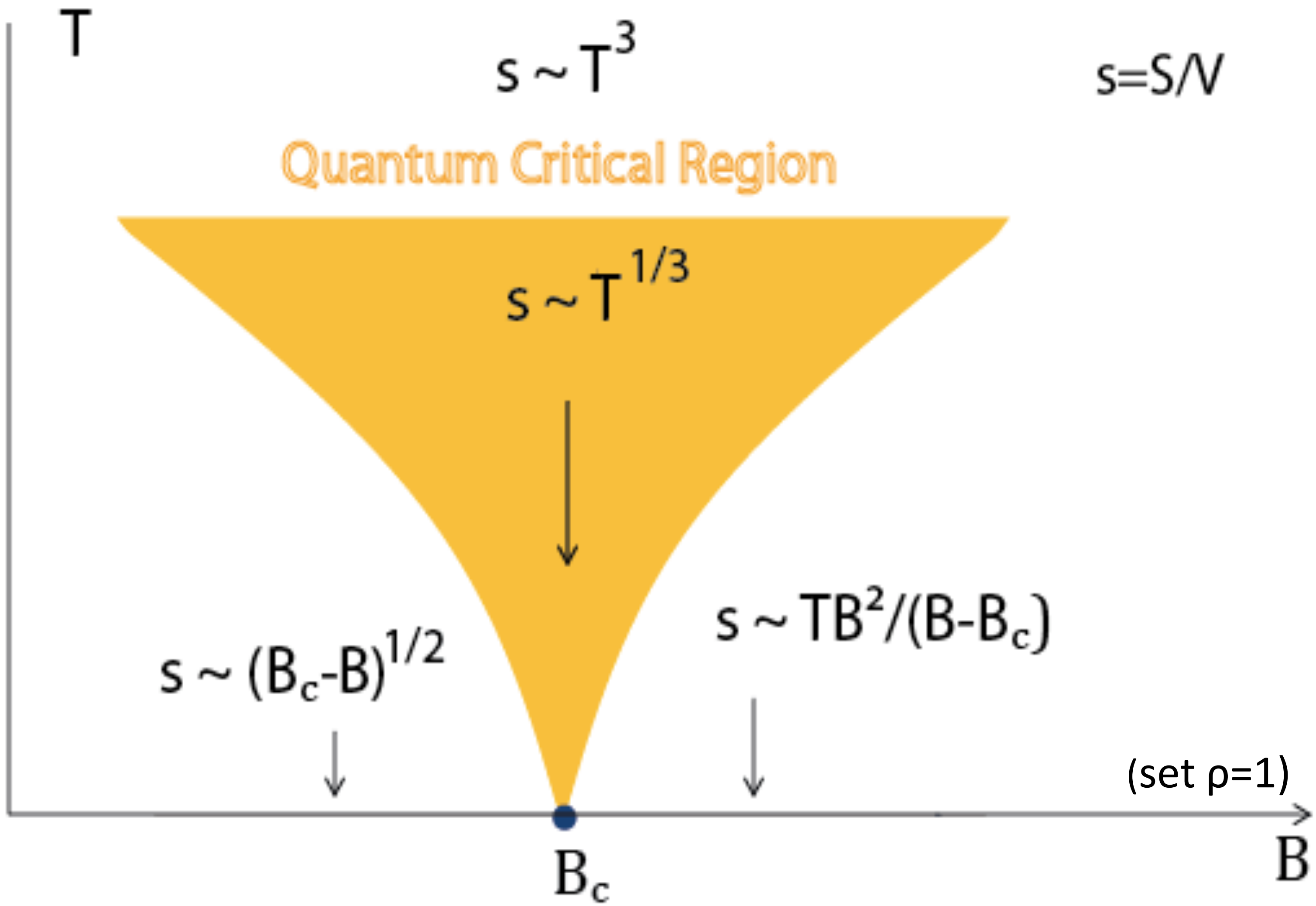
$T^2 << B$ lowest Landau level dominates at low energy



$B << T^2$ entropy density $S/V \approx T^3$

$T^2 << B$ entropy density $S/V \approx BT$ (free Fermions)

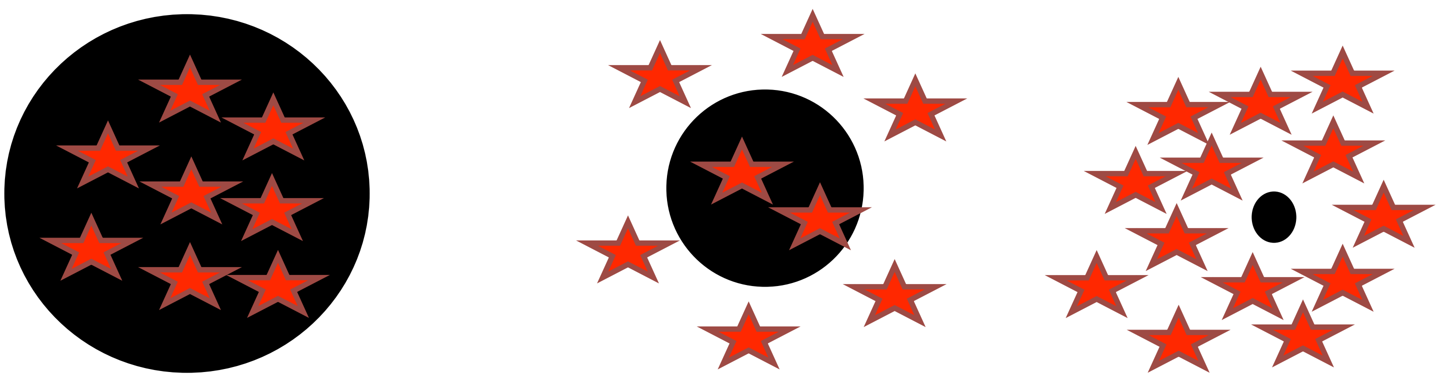
Charge Density & Magnetic field



(ED, Kraus)

Gravitational picture

Balancing charges of carriers and black hole



$B=0$ $0 < B < B_c$ $B_c < B$

Quantum phase transition driven by charge expulsion from black hole at T=0 (extending to crossover for low T)

Meta-magnetic transitions

= quantum phase transitions across non-zero B

- Can be realized experimentally in layered materials
  - A favorite is $Sr_3 Ru_2 O_7$

- Exponent 1/3 (familiar from Hertz – Millis)
- Prediction $s/T \approx 1/(B-B_c)$
  - Not explained by Fermi liquid theory
- $B < B_c$ phase with non-zero s at T=0 ???

Entropy landscape in $Sr_3 Ru_2 O_7$

Rost, Perry, Mercure, Mackenzie, Grigera (2009)

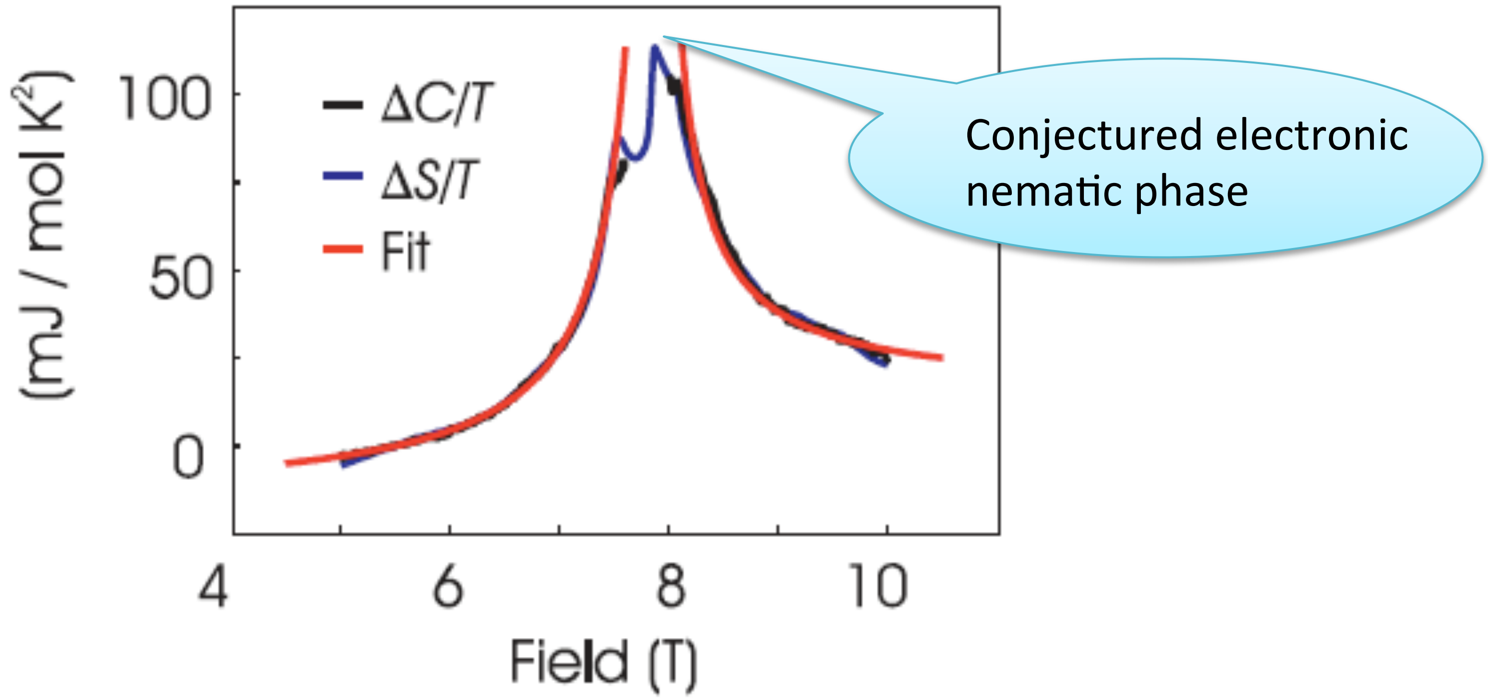
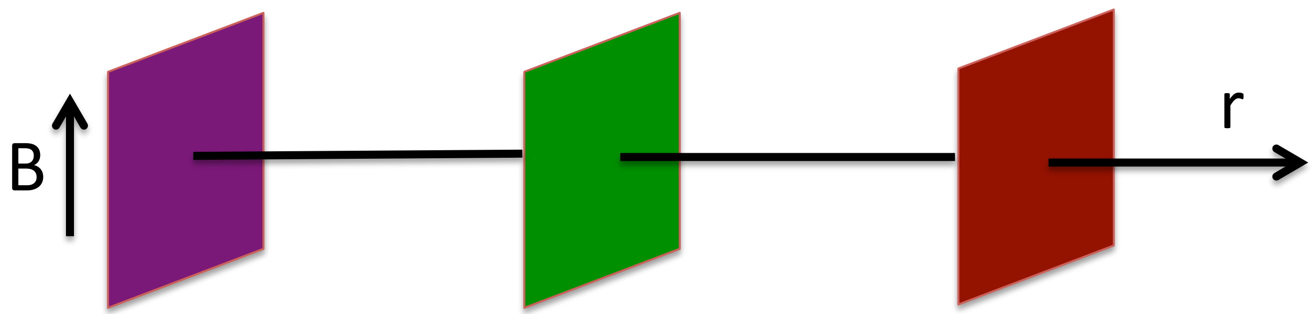


Fig. 4. (**A**) Entropy divided by temperature as a function of applied field and temperature for $Sr_3Ru_2O_7$. (**B**) Both $\Delta S/T$ (blue) and the specific heat, $\Delta C/T$ (black), diverge as $[(H - H_c)/H_c]^{-1}$ outside the cut

What do we learn ?

- ✓ Gauge-gravity prediction $s/T \approx 1/(B-Bc)$ good
  - – divergence is taken over by less entropic phase
  - – Exponent 1/3 is inaccessible experimentally
- Gauge-gravity predicts $S > 0$ at $T = 0$ for $B < B_c$
  - – Presumably unstable phase
- Experimentally, near $B = B_c$ nematic phase
  - – Broken translational/rotational symmetry
- ➢ Can gauge/gravity produce nematic phase ?

Broken translational invariance



Lift restriction of translation invariant solutions at fixed r

- Simple way of breaking translations by p-wave

  $z \to z + d \qquad x \to x\cos(kd) - y\sin(kd)$

  $\qquad\qquad\quad\; y \to x\sin(kd) + y\cos(kd)$

- For B=0 such solutions exist

  and stabilizes the S>0 phase at T=0

  (Nakamura, Ooguri, Park; Donos, Gauntlett ….)

- For $B \approx B_c$, under investigation (ED, Kraus)

Organizing Universal IR Behavior

- As always … guided by symmetry …
  - Symmetry over long distances of nematics/smectics
  - Various degrees of broken translations/rotations
- For homogeneous materials
  - Near-horizon gravity solution homogeneous spaces
  - Bianchi classification -- cosmology ! ! (Landau & Lifschitz)
  - Systematic approach initiated by

(Iizuka, Kachru, Kundu, Narayan, Sircar, Trivedi 2012)

Conclusions

- Gauge-gravity duality provides promising tool
  - Variety of strongly correlated CM systems
    - Quantum criticality, superfluidity, superconductivity
    - Including broken translation/rotation invariance, lattices
    - Transport problems in real time, plasmas, hydrodynamics
    - Quantum quenches, entanglement entropy
  - Ambitious goal
    - Understand/classify all strong coupling fixed points

Thank You