Five-dimensional gauge theory via holography

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Introduction

- **Standard QFT textbooks tell us**:
  - *gauge theory is renormalizable only in four or fewer dimensions*

- **Dynamics of branes in M-theory and Type II suggest otherwise**:
  - *Six-dimensional superconformal theories with 32 or 16 supercharges;*
  - *Five-dimensional superconformal fixed points with 16 supercharges*
  - *Possess Coulomb branch and Higgs branch deformations by relevant operators to non-conformal theories*

Overview

- **Field theory approach** (review)
  - *Existence of finite Coulomb branch and super-conformal phase*
- **String/branes approach** (review)
  - *D5/NS5 webs in Type IIB*
- **Holographic approach to the super-conformal phase**
  - *Type IIB supergravity on* $AdS_6 \times S^2$ *warped over Riemann surface* Σ
  - *Obtain exact local solutions to the BPS equations for 16 supersymmetries*
  - *Construct global solutions*
  - *Open problems*

Bibliography

Key earlier work

- *Five-dimensional SUSY Field Theories, Non-trivial Fixed Points, and String Dynamics*, N. Seiberg, hepth/9608111;
- *Five-Dimensional Supersymmetric Gauge Theories and Degenerations of Calabi-Yau Spaces*, K. Intriligator, D.R. Morrison, N. Seiberg, hepth/9702198;

Our papers

- *Warped $AdS_6 \times S^2$ in Type IIB supergravity I: Local Solutions*, ED, Michael Gutperle, Andreas Karch, Christoph F. Uhlemann, arXiv:1606.01254;
- *Holographic duals for five-dimensional superconformal quantum field theories*, ED, Michael Gutperle, Christoph F. Uhlemann, arXiv:1611.09411;
- *Warped $AdS_6 \times S^2$ in Type IIB supergravity II: Global Solutions and five-brane webs*, ED, Michael Gutperle, Christoph F. Uhlemann, arXiv:1703.08186.

Five-dimensional gauge theory

- Five-dimensional gauge theory (e.g. $SU(N)$ gauge group)

$$\mathcal{L} \sim g^{-2}\,\mathrm{tr}(F^2) + \frac{c}{24\pi^2}\mathrm{tr}(A \wedge F \wedge F + \cdots)$$

  - $[g^{-2}] =$ mass and hence perturbatively non-renormalizable;
  - c quantized in integers by gauge invariance;

  - View $\mathrm{tr}(F^2)$ as relevant deformation on marginal CS term ?
  - Theory naturally contains solitonic states
    - * 4-dim instantons $\longrightarrow$ 5-dim particles, $U(1)_I$ charge $J = \mathrm{tr}(F \wedge F)$
    - * 4-dim magnetic monopoles $\longrightarrow$ 5-dim strings.

- cfr Three-dimensional gauge theory

$$\mathcal{L} \sim g^{-2}\,\mathrm{tr}(F^2) + \frac{c}{2\pi}\mathrm{tr}(A \wedge F + \cdots)$$

  - g^2 has dimensions of mass, hence perturbatively renormalizable;
  - c quantized in integers by gauge invariance;
  - View $\mathrm{tr}(F^2)$ as irrelevant deformation on marginal CS term.

Five-dimensional supersymmetry

- Minimal Poincaré supersymmetry in five dimensions
  - has 2 supersymmetry spinor generators = 8 real supersymmetries
  - and thus $SU(2)_R$ symmetry
  - under S^1 compactification reduces to $\mathcal{N} = 2$ in $d = 4$.

- Supermultiplets
  - gauge multiplet $\mathcal{A} = (A_\mu, \lambda_\alpha, \phi)$ with ϕ a real scalar;
  - hypermultiplet $\mathcal{H} = (H_a, \psi)$ with H_a four real scalars;

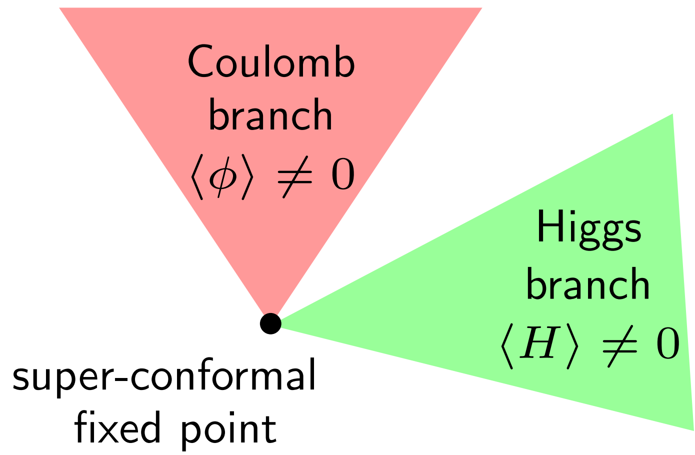
- Super-conformal symmetry
  - the conformal algebra in $d \geq 3$ dimensions is $SO(2, d)$
  - the superconformal algebra contains $SO(2, d)$, the R-symmetry algebra, and fermionic generators which are spinors under $SO(2, d)$

$d = 3$	$OSp(2m\|4)$	$SO(2, 3) = Sp(4, \mathbb{R}), \ m = 1, 2, 3, 4$
$d = 4$	$SU(2, 2\|m)$	$SO(2, 4) = SU(2, 2), \ m = 1, 2, 3, 4$
$d = 5$	$F(4)$	$SO(2, 5) \oplus SU(2)$ max bosonic subalgebra
$d = 6$	$OSp(8^*\|2m)$	$SO(2, 6) = SO(8^*), \ m = 1, 2$

  - maximal 32 supersymmetries in $d = 3, 4, 6$ but only 16 in $d = 5$.

The Coulomb and Higgs branches

- When hypermultiplets are included, both branches exist,
  - Coulomb branch: gauge scalars acquire vevs $\langle \phi \rangle \neq 0$
  - Higgs branch: hypermultiplet scalars acquire vevs $\langle H \rangle \neq 0$



- Reach super-conformal fixed point via Coulomb branch (Seiberg, 1996)
  - generically $SU(N) \to U(1)^{N-1}/\mathrm{Weyl}$
  - $U(1)$ gauge supermultiplets $\mathcal{A}^i = (A^i_\mu, \lambda^i_\alpha, \phi^i)$
    with $i = 1, \cdots, N, \ \sum_i \mathcal{A}^i = 0$

The pre-potential

- Dynamics in the Coulomb branch is governed by a pre-potential $\mathcal{F}(\mathcal{A}^i)$
  - just as for $d=4$, $\mathcal{N}=2$ (Seiberg-Witten 1994)
  - bosonic part of the Lagrangian dictated by supersymmetry

$$\mathcal{L} \sim \sum_{i,j} \partial_i \partial_j \mathcal{F}(\phi) \Big(F^i F^j + \partial \phi^i \partial \phi^j \Big) + \sum_{i,j,k} \partial_i \partial_j \partial_k \mathcal{F}(\phi) \Big(A^i \wedge F^j \wedge F^k + \cdots \Big)$$

– Gauge invariance $A^i \to A^i + d\theta^i$ requires $\partial^3 \mathcal{F}$ to be constant.
– Hence the pre-potential is at most cubic in $\phi^i, \mathcal{A}^i$.
– Quantum corrections at one-loop only are due to massive BPS states,
– no instanton contributions (in contrast with $d=4$, $\mathcal{N}=2$).

- Exact pre-potential for $SU(N)$ with N_f hypermultiplets in the $\mathbf{N}$ of $SU(N)$

$$\mathcal{F}(\phi) = \frac{1}{2g_0^2} \sum_i \phi_i^2 + \frac{c}{6} \sum_i (\phi_i)^3 + \frac{1}{6} \sum_{i<j} |\phi_i - \phi_j|^3 - \frac{1}{12} \sum_{f=1}^{N_f} \sum_i |\phi_i + m_f|^3$$

– the bare coupling g_0^2 is a UV cutoff, m_f are hypermultiplet masses.

Dynamics on the Coulomb branch

- Regularity requires the gauge kinetic energy to have positive sign,
  - $\partial_i \partial_j \mathcal{F}$ must be positive for $\phi \in \mathbb{R}^{N-1}/\mathrm{Weyl}$
- For $SU(2)$ gauge group $\phi = \phi_1 = -\phi_2$, and N_f hypermultiplets,

$$\frac{1}{g^2(\phi)} = \frac{1}{g_0^2} + 2\,|\phi| - \frac{1}{4}\sum_{f=1}^{N_f} |\phi - m_f| \qquad\qquad \frac{1}{g^2(\phi)} = \partial^2 \mathcal{F}(\phi)$$

- Requiring $g^2(\phi) > 0$ for regularity,
  - we can remove the UV cutoff, $g_0^2 \to \infty$, provided $N_f \leq 7$,
  - leaving a UV finite theory on the Coulomb branch.
  - Superconformal fixed point as $\langle \phi \rangle, m_f \to 0$ is strongly coupled.
  - Global $SO(2N_f)$ symmetry of hypermultiplets, and $U(1)_I$ of "instantons" enhanced to E_{N_f+1} global symmetry $(E_8, E_7, E_6, SO(10), SU(5), \cdots)$
- Extends to $SU(N)$ with N_f hypermultiplets provided $N_f \leq 2N$.

Supersymmetric field theories from branes

- Standard cases have maximal supersymmetry
  - 16 Poincaré supercharges
  - in the near-horizon limit enhanced to 32 conformal supercharges

dim	theory	brane	near-horizon geometry	asymptotic symmetry
d=3	M-theory	M2	$AdS_4 \times S^7$	$SO(2,3) \times SO(8)$
d=4	Type IIB	D3	$AdS_5 \times S^5$	$SO(2,4) \times SO(6)$
d=6	M-theory	M5	$AdS_7 \times S^4$	$SO(2,6) \times SO(5)$

- For $d = 5$, superconformal $F(4)$ is unique and has 16 supercharges (8 Poincaré)

- Brane approaches to five-dimensional gauge theory
  - D4 probe brane and parallel D8 branes in massive Type IIA
    (Seiberg, 1996) and (Brandhuber, Oz 1999)
  - D5 intersecting NS5 branes in Type IIB
    (Aharony, Hanany 1997)

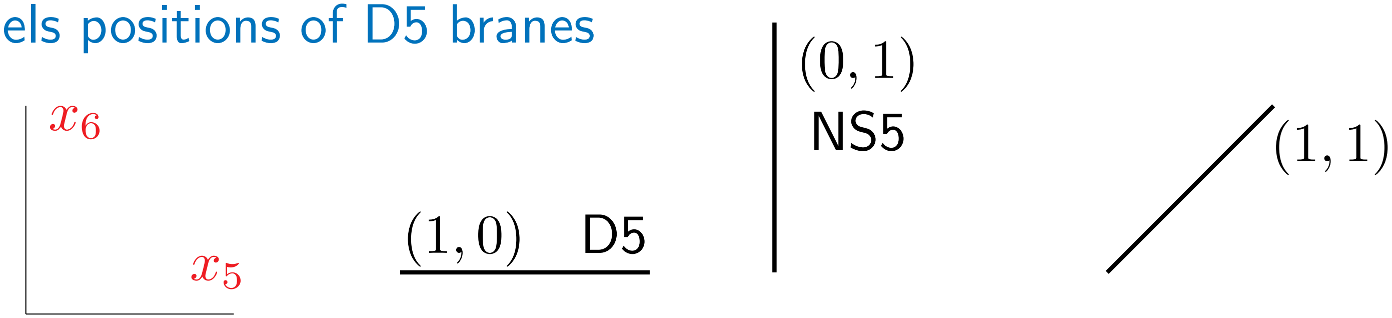
- M-theory on 6-dim Calabi-Yau approach to five-dimensional fixed points
  (Morrison, Seiberg, 1996)

Five-branes in Type IIB string theory

- D5 and NS5 branes intersecting along a five-dimensional space-time
  - Poincaré $ISO(1,4)$ invariant along 01234 parallel directions
  - $SO(3)$ invariant along 789-transverse directions
  - has 8 Poincaré supersymmetries

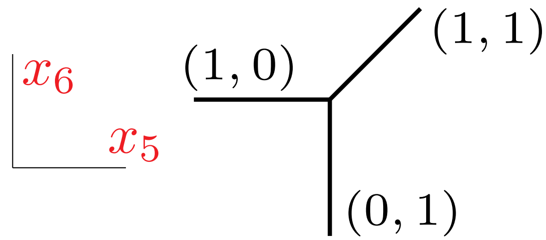
branes	0	1	2	3	4	5	6	7	8	9
D5	×	×	×	×	×	×				
NS5	×	×	×	×	×		×			

- D5 and NS5 transform under $SL(2,\mathbb{Z})$ duality of Type IIB (Schwarz 1995)
  - (p,q) five-branes with $p,q \in \mathbb{Z}$
  - x_5 labels positions of NS5 branes,
  - x_6 labels positions of D5 branes

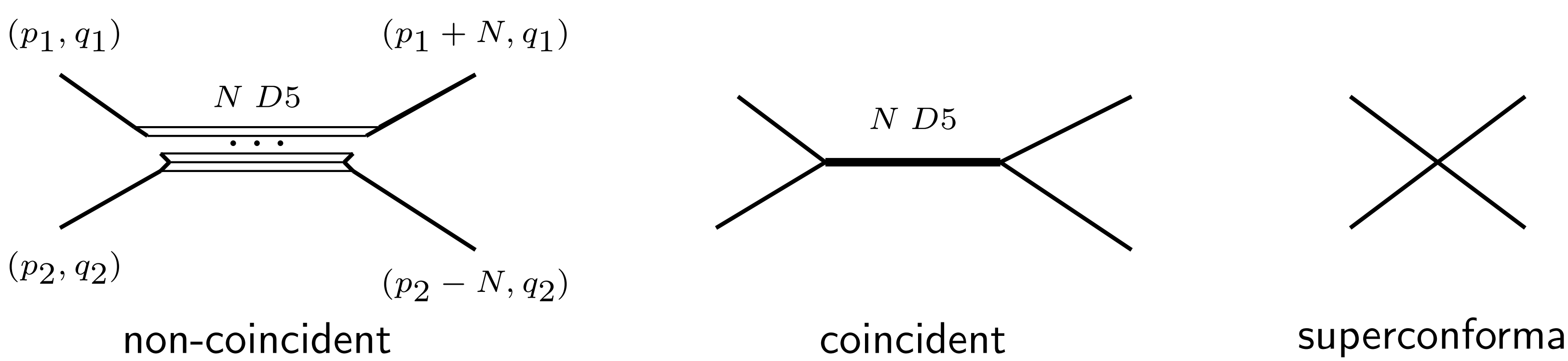


(p,q) brane webs

- (p,q)-brane intersections conserve p,q-charges due to $SL(2,\mathbb{Z})$ symmetry



- N parallel D5 branes suspended between two semi-infinite branes
  - non-coincident: $U(1)^{N-1}$ gauge theory plus massive W-bosons
  - coincident: $SU(N)$ gauge theory
  - superconformal: web collapses to a single point



Near-horizon limit

- Take the near-horizon limit of a (p, q) web configuration
  - with a large number N of coincident D5 branes

branes	0	1	2	3	4	5	6	7	8	9
D5	×	×	×	×	×	×				
NS5	×	×	×	×	×		×			

  - radial coordinate in 789 direction combines with 01234 to AdS_6
  - remaining angular directions of 789 give S^2
  - with combined isometries $SO(2,5) \times SO(3)$

- Total space-time geometry

$$AdS_6 \times S^2 \times \Sigma$$

  - where $AdS_6 \times S^2$ is warped over the two-dimensional surface Σ
  - Σ contains the structure of the web in the near-horizon limit

- Our approach: obtain the Type IIB supergravity solutions directly
  - several earlier attempts (no physically interesting solutions)

    Lozano et al, 2012; Apruzzi et al, 2014; Kim et al 2015; O'Colgain et al 2015

Type IIB supergravity

- The fields of Type IIB sugergravity are

g_{MN}	metric		
B	axion/dilaton	$P, Q \sim dB$	(contains χ, Φ)
C_2	complex 2-form	G	(contains NSNS, RR)
C_4	real 4-form	F_5	$\star F_5 = F_5$
ψ_M	gravitino	Weyl spinor	
λ	dilatino	Weyl spinor	

- Type IIB supergravity is invariant under global $SL(2,\mathbb{R}) = SU(1,1)$
  - Einstein-frame metric and F_5 are invariant,
  - dilaton/axion B in coset $SU(1,1)/U(1)$, complex C_2 transforms linearly,

$$B \to \frac{uB + v}{\bar{v}B + \bar{u}} \qquad C_2 \to uC_2 + v\bar{C}_2 \qquad |u|^2 - |v|^2 = 1$$

- Bianchi identities and field equations.

Supersymmetric solutions and BPS equations

- Susy variations in Type IIB at vanishing Fermi fields

$$\delta\lambda = iP\cdot\Gamma\,\mathcal{B}^{-1}\varepsilon^* - \frac{i}{4}(G\cdot\Gamma)\varepsilon$$

$$\delta\psi_M = D_M\varepsilon + \frac{i}{4}(F_5\cdot\Gamma)\,\Gamma_M\varepsilon - \frac{1}{16}\Big(\Gamma_M(G\cdot\Gamma) + 2(G\cdot\Gamma)\Gamma_M\Big)\mathcal{B}^{-1}\varepsilon^*$$

– Γ_M are Dirac matrices, $(\Gamma_M)^* = \mathcal{B}\Gamma_M\mathcal{B}^{-1}$ effects charge conjugation.

– A configuration is supersymmetric if $\delta\psi_M = \delta\lambda = 0$ has solutions with $\varepsilon \neq 0$
– A configuration is half-BPS if there are 16 linearly independent solutions ε

- BPS equations remind of Lax equations in integrable systems

field equations $\Leftrightarrow$ integrability of system of linear differential eqs

– with 32 susys, BPS eqs imply all Bianchi and field equations;
– with ≥ 28 susy, interesting results (Gran, Gutowski, Papadopoulos)
– with 16 susys, BPS eqs plus some Bianchi identities imply all the field eqs;

The supergravity Ansatz

- The $SO(2,5) \times SO(3)$ symmetry dictates the space-time structure,

$$AdS_6 \times S^2 \text{ warped over a Riemann surface } \Sigma$$

- The metric and flux fields are restricted by symmetry,

$$\begin{aligned} ds^2 &= f_6^2 \, d\hat{s}^2_{AdS_6} + f_2^2 \, d\hat{s}^2_{S^2} + ds^2_\Sigma \\ F_3 &= g_a \, e^a \wedge e^6 \wedge e^7 \\ P &= p_a \, e^a \\ Q &= q_a e^a \\ F_5 &= 0 \end{aligned}$$

  – $d\hat{s}^2_{AdS_6}$ and $d\hat{s}^2_{S^2}$ have unit radius "round" metrics;

  – e^A is orthonormal frame, $A = 6,7$ for S^2 and $A = a = 8,9$ for Σ

  – $ds^2_\Sigma = e^a \otimes e^b \delta_{ab}$ with $a, b = 8, 9$.

Reducing the BPS equations

- Use Killing spinors on $AdS_6 \times S^2$ as basis for the susy parameter ε,

$$\varepsilon = \sum_{\eta_1,\eta_2} \chi^{\eta_1,\eta_2} \otimes \zeta_{\eta_1,\eta_2}$$

  - χ^{η_1,η_2} fixed basis of Killing spinors, $\eta_1 = \pm$ and $\eta_2 = \pm$ independently;
  - ζ_{η_1,η_2} are 2-component spinors on Σ.

- The BPS equations reduce to a system of 4 spinor equations,

$$\begin{aligned}
0 &= 4p_a \gamma^a \gamma^9 \zeta^* - g_a \tau^3_{(2)} \gamma^a \zeta \\
0 &= -\frac{i}{2f_6} \tau^2_{(1)} \otimes \tau^1_{(2)} \zeta + \frac{D_a f_6}{2f_6} \gamma^a \zeta - \frac{1}{16} g_a \tau^3_{(2)} \gamma^a \gamma^9 \zeta^* \\
0 &= \frac{1}{2f_2} \tau^2_{(2)} \zeta + \frac{D_a f_2}{2f_2} \gamma^a \zeta + \frac{3}{16} g_a \tau^3_{(2)} \gamma^a \gamma^9 \zeta^* \\
0 &= \left(D_a + \frac{i}{2} \omega_a \sigma^3 - \frac{i}{2} q_a \right) \zeta + \frac{3}{16} g_a \tau^3_{(2)} \gamma^9 \zeta^* - \frac{1}{16} g_b \tau^3_{(2)} \gamma_a{}^b \gamma^9 \zeta^*
\end{aligned}$$

  - Derivative D_a and connection ω_a are defined with respect to the frame e^a,
  - $\tau_{(1,2)}$ are Pauli matrices acting on indices $\eta_{1,2}$.

Decoupling the reduced BPS equations

- Algebraic methods used to restrict range of ζ (ED, Estes, Gutperle 2007)

$$\begin{aligned} \bar\alpha &= \zeta_{+++} = -\zeta_{--+} = -i\nu\zeta_{+-+} = +i\nu\zeta_{-++} \qquad \nu^2 = 1 \\ \bar\beta &= \zeta_{---} = +\zeta_{++-} = -i\nu\zeta_{-+-} = -i\nu\zeta_{+--} \end{aligned}$$

- The radii f_6 and f_2 may be obtained algebraically in terms of α, β,

$$f_6 = 3(|\alpha|^2 + |\beta|^2) \qquad\qquad f_2 = -\nu(|\alpha|^2 - |\beta|^2)$$

  – Choose local complex coordinates $(w, \bar w)$ with $e^z = e^8 + ie^9 = \rho\, dw$
  – Use Bianchi identities to express the fields $p_z, q_z, p_{\bar z}, q_{\bar z}$ in terms of B

- Two of the four differential equations may be integrated exactly,

$$\begin{aligned} \rho\bar\alpha^2 &= f(\kappa_+ + B\,\kappa_-) \qquad & \kappa_\pm &= \partial_w \mathcal{A}_\pm \\ \rho\bar\beta^2 &= f(\bar B\,\kappa_+ + \kappa_-) \qquad & f^{-2} &= 1 - |B|^2 \end{aligned}$$

  – where $\mathcal{A}_\pm$ are arbitrary locally holomorphic functions on Σ.

Secret to integrability

- The remaining reduced equations for $B, \bar{B}, \rho$ are as follows

$$
\begin{aligned}
2\,\partial_w \ln \rho - f^2(\partial_w \bar{B})\,\frac{\kappa_+ + B\kappa_-}{\bar{B}\kappa_+ + \kappa_-} - 2f^2(\partial_w \bar{B})\,e^{+i\vartheta} &= \frac{\bar{B}\partial_w\kappa_+ + \partial_w\kappa_-}{\bar{B}\kappa_+ + \kappa_-} \\
2\,\partial_w \ln \rho - f^2(\partial_w B)\,\frac{\bar{B}\kappa_+ + \kappa_-}{\kappa_+ + B\kappa_-} - 2f^2(\partial_w B)\,e^{-i\vartheta} &= \frac{\partial_w\kappa_+ + B\partial_w\kappa_-}{\kappa_+ + B\kappa_-} \\
(\partial_w B)\,\frac{(\bar{\kappa}_+ + \bar{B}\bar{\kappa}_-)^{\frac{3}{2}}}{(B\bar{\kappa}_+ + \bar{\kappa}_-)^{\frac{1}{2}}} - (\partial_w \bar{B})\,\frac{(B\bar{\kappa}_+ + \bar{\kappa}_-)^{\frac{3}{2}}}{(\bar{\kappa}_+ + \bar{B}\bar{\kappa}_-)^{\frac{1}{2}}} + \frac{2\rho^2}{3f^3} &= 0
\end{aligned}
$$

  – where the phase angle ϑ is defined by,

$$
e^{2i\vartheta} = \left(\frac{\kappa_+ + B\kappa_-}{\bar{\kappa}_+ + \bar{B}\bar{\kappa}_-}\right)\left(\frac{B\bar{\kappa}_+ + \bar{\kappa}_-}{\bar{B}\kappa_+ + \kappa_-}\right)
$$

- This system is actually solvable,
  – Effectively a Lax system on Σ, and thus integrable in principle,
  – Three fields $(B, \bar{B}, \rho)$ version of the sine-Gordon-Liouville-Toda type

Local solutions to the BPS equations

- Metric components of the solution are given as follows,

$$\rho^4 = \frac{R(1+R)(\kappa^2)^3}{|\partial_w \mathcal{G}|^2(1-R)} \qquad f_2^2 = \frac{\kappa^2(1-R)}{\rho^2(1+R)} \qquad f_6^2 = \frac{9\kappa^2(1+R)}{\rho^2(1-R)}$$

  – in terms of the following combinations,

$$\kappa^2 = -|\partial_w \mathcal{A}_+|^2 + |\partial_w \mathcal{A}_-|^2 \qquad R + \frac{1}{R} = 2 + \frac{6\kappa^2 \mathcal{G}}{|\partial_w \mathcal{G}|^2}$$

$$\partial_w \mathcal{B} = \mathcal{A}_+ \partial_w \mathcal{A}_- - \mathcal{A}_- \partial_w \mathcal{A}_+ \qquad \mathcal{G} = |\mathcal{A}_+|^2 - |\mathcal{A}_-|^2 + \mathcal{B} + \bar{\mathcal{B}}$$

- $SU(1,1)$ symmetry of Type IIB acts naturally,

$$B \to \frac{uB+v}{\bar{v}B+\bar{u}} \qquad \begin{pmatrix} \mathcal{A}_+ \\ \mathcal{A}_- \end{pmatrix} \to \begin{pmatrix} u & -v \\ -\bar{v} & \bar{u} \end{pmatrix} \begin{pmatrix} \mathcal{A}_+ \\ \mathcal{A}_- \end{pmatrix} \qquad |u|^2 - |v|^2 = 1$$

  – manifestly leaves $\kappa^2, \mathcal{G}$ and thus the Einstein frame metric invariant

- Positivity of metric functions f_6^2, f_2^2, ρ^4 when $\kappa^2,\ \mathcal{G} > 0$ choosing $R < 1$.

Strategy for global solutions

- Summary of the associated mathematical problem
  - Riemann surface Σ of unknown type (genus ? boundaries ?)
  - Locally holomorphic functions $\mathcal{A}_+, \mathcal{A}_-$ on Σ
    - ⋆ with linear transformation law under $SU(1,1)$ symmetry of Type IIB
    - ⋆ subject to positivity conditions

$$0 \quad < \quad \kappa^2 = -|\partial_w \mathcal{A}_+|^2 + |\partial_w \mathcal{A}_-|^2$$
$$0 \quad < \quad \mathcal{G} = |\mathcal{A}_+|^2 - |\mathcal{A}_-|^2 + \mathcal{B} + \bar{\mathcal{B}}$$

- No (regular) solutions when Σ is compact without boundary,

$$\partial_{\bar w} \partial_w \mathcal{G} = -\kappa^2 \qquad \Longrightarrow \qquad \int_\Sigma \kappa^2 = 0$$

- The boundary $\partial\Sigma$ of Σ has vanishing S^2 radius

$$\partial\Sigma : \quad f_2 \to 0 \qquad f_6 \neq 0$$

  - $\partial\Sigma$ is not a boundary of the solution's space-time manifold,
  - $\partial\Sigma$ corresponds to S^2 slice of S^3 cycle,
  - requires $\kappa^2 = \mathcal{G} = 0$ on $\partial\Sigma$.

Inspiration from Electro-statics

- Holomorphic $SU(1,1)$-vector bundles give unproductive hint.

- Map this onto an electro-statics problem.
  - Consider the locally meromorphic ratio λ on Σ (it can have poles)

$$\lambda = \frac{\partial_w \mathcal{A}_+}{\partial_w \mathcal{A}_-} \qquad\qquad \kappa^2 = -|\partial_w \mathcal{A}_+|^2 + |\partial_w \mathcal{A}_-|^2$$

    ⋆ in the interior of Σ the condition $\kappa^2 > 0$ requires $|\lambda|^2 < 1$
    ⋆ on the boundary of Σ the condition $\kappa^2 = 0$ requires $|\lambda|^2 = 1$

  - Consider the "electro-static potential"

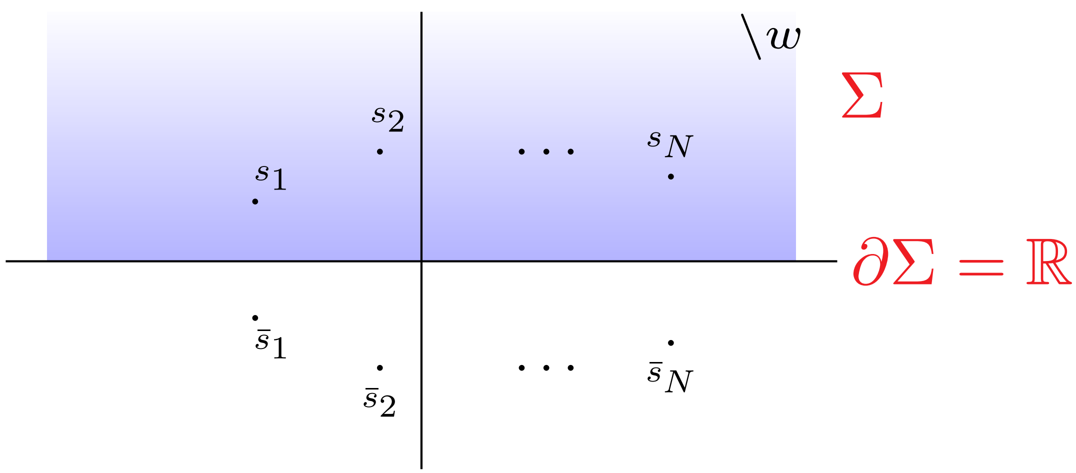
$$\Phi = -\ln |\lambda|^2$$

    ⋆ Φ is real harmonic on Σ
    ⋆ $\Phi > 0$ in the interior of Σ, and $\Phi = 0$ on the boundary of Σ

- Place an array of positive electric charges in the interior of Σ and opposite image charges in the mirror image of Σ

Σ of genus zero and one boundary component

- With a single boundary component, and genus zero,
  - $\partial\Sigma$ may be mapped onto the real line
  - Σ may be mapped onto the upper half plane



- The general electro-static solution is immediate

$$\Phi(w) = -\ln|\lambda|^2 = -\sum_{n=1}^{N} q_n \Big(\ln|w - s_n|^2 - \ln|w - \bar{s}_n|^2 \Big) \qquad q_n > 0$$

- for arbitrary N, q_n, s_n.

Solving for the differentials

- Regularity of the meromorphic function λ requires $q_n = 1$ for all n,

$$\lambda(w) = \prod_{n=1}^{N} \frac{w - s_n}{w - \bar{s}_n}$$

- Assuming $\partial_w \mathcal{A}_\pm$ meromorphic, $\partial_w \mathcal{A}_+ = \lambda \partial_w \mathcal{A}_-$ and regularity require,

$$\partial_w \mathcal{A}_+ = \frac{1}{R(w)} \prod_{n=1}^{N} (w - s_n)$$

$$\partial_w \mathcal{A}_- = \frac{1}{R(w)} \prod_{n=1}^{N} (w - \bar{s}_n)$$

  - $R(w)$ is polynomials with only real roots r_ℓ

$$R(w) = \prod_{\ell=1}^{\deg R} (w - r_\ell)$$

  - real zeros are also allowed but may be viewed as the limit of $\mathrm{Im}(s_n) \to 0$
  - regularity at ∞ requires $\deg R = N + 2$

Satisfying the regularity conditions

- Alternative form of $\partial_w \mathcal{A}_\pm$,

$$\partial_w \mathcal{A}_\pm(w) = \sum_{\ell=1}^{N+2} \frac{Z_\pm^\ell}{w - r_\ell} \qquad\qquad Z_+^\ell = (Z_-^\ell)^* = \frac{1}{P'(r_\ell)} \prod_{n=1}^{N} (r_\ell - \bar{s}_n)$$

– allows us to integrate up to $\mathcal{A}_\pm$,

$$\mathcal{A}_\pm(w) = \sum_{\ell=1}^{N+2} Z_\pm^\ell \ln(w - r_\ell)$$

– and to obtain $\mathcal{B}$ in terms of "dilogarithm integrals"

$$\mathcal{B}(w) = \sum_{\ell,\ell'=1}^{N+2} \left(Z_+^\ell Z_-^{\ell'} - Z_+^{\ell'} Z_-^\ell \right) \int_{w_0}^{w} dw \, \frac{\ln(w - r_\ell)}{w - r_{\ell'}}$$

– judicious choice of branch cuts allows one to show that

$$\mathcal{G} = |\mathcal{A}_+|^2 - |\mathcal{A}_-|^2 + \mathcal{B} + \bar{\mathcal{B}}$$

  - obeys $\mathcal{G} = 0$ on the boundary of Σ
  - obeys $\mathcal{G} > 0$ in the interior of Σ

Asymptotics near pole = near (p,q) five-brane

- The solution is regular everywhere on Σ, except at the poles r_ℓ

$$w = r_\ell + u\, e^{i\theta}$$

- The dilaton diverges and the Einstein-frame metric becomes,

$$ds^2 = (-\ln u) d\hat s^2_{AdS_6} + |Z^\ell_+ - Z^\ell_-| \left(\frac{du^2}{u^2} + d\hat s^2_{S^3} \right)$$

  – AdS_6 expands to infinite radius, by rescaling tends to $\mathbb{R}^6$;
  – (p,q)-charges at the pole given by $p_\ell = \mathrm{Re}(Z^\ell_+)$ and $q_\ell = -\mathrm{Im}(Z^\ell_+)$

- Stack of N coincident NS5 branes produces string frame metric and dilaton,

$$ds^2 = dx^\mu dx_\mu + e^{2\Phi} d\mathbf{y}^2 \qquad\qquad e^{2\Phi(\mathbf{y})} = e^{2\Phi(\infty)} + N/\mathbf{y}^2$$

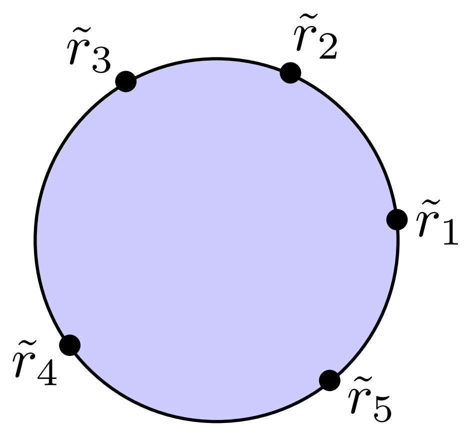
  – x^μ along 5-brane, $\mathbf{y}$ perpendicular to 5-brane, near-horizon $u^2 = \mathbf{y}^2 \to 0$

$$ds^2 \sim dx^\mu dx_\mu + \frac{du^2}{u^2} + ds^2_{S^3} \qquad\qquad e^{2\Phi(\mathbf{y})} \sim N/u^2$$

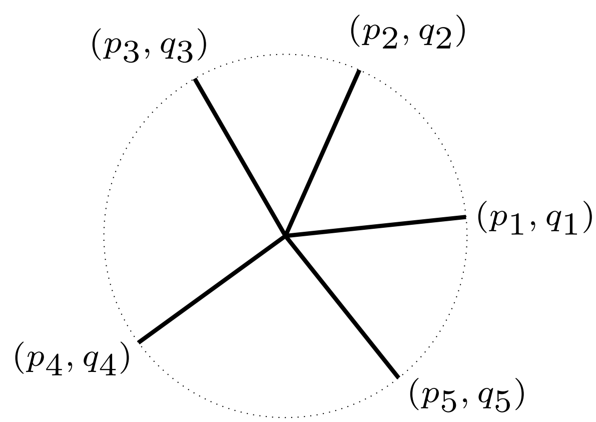
  – agrees with behavior near the poles of our solutions

Poles represent semi-infinite "heavy" branes

- Conformally map the upper half plane to the unit disc;
  - real axis to unit circle
  - points $r_\ell \in \mathbb{R}$ to points $\tilde{r}_\ell$ on unit circle



Riemann surface Σ



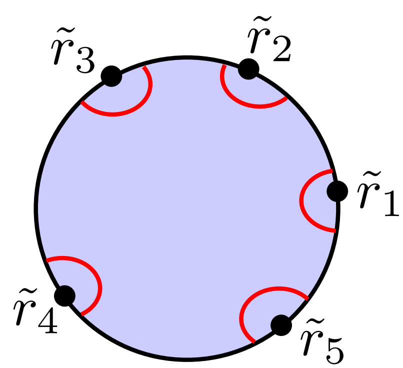
(p, q) five-brane web

Correlators holographically ?

- Key motivation for obtaining our Type IIB supergravity solutions
  - access the superconformal phase of five-dimensional gauge theories
  - compute operator dimensions and correlators

- For standard cases, asymptotically region has enhanced symmetry
  - eg asymptotically $SU(2,2|4)$ for asymptotic $AdS_5 \times S^5$
  - In five dimensions superconformal algebra $F(4)$ throughout

- The "heavy" effectively six-dimensional branes are part of the solution (as poles)
  - Effects of warping persist to the holographic boundary
  - For five-dim holography one must prevent access to six-dim regions
  - impose boundary conditions on red "walls" ?



Outlook

- We constructed exactly a wealth of $AdS_6 \times S^2 \times \Sigma$ solutions in Type IIB
  - regular except for expected asymptotics of "heavy" (p,q) branes,
  - precise matching of parameters in brane and supergravity constructions,
  - solutions with D7-branes (ED, Gutperle, Uhlemann arXiv:1706.00433)
  - Local solutions to the "double analytic continuation" $AdS_2 \times S^6 \times \Sigma$
- Dynamics of five-dimensional superconformal gauge theories
  - spectrum of operator dimensions around the solutions,
  - do these solutions have exceptional global symmetries $E_8, E_7, E_6, \cdots$?