Exact M-theory Solutions, Branes and Superalgebras

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Exact Solvability and Symmetry Avatars

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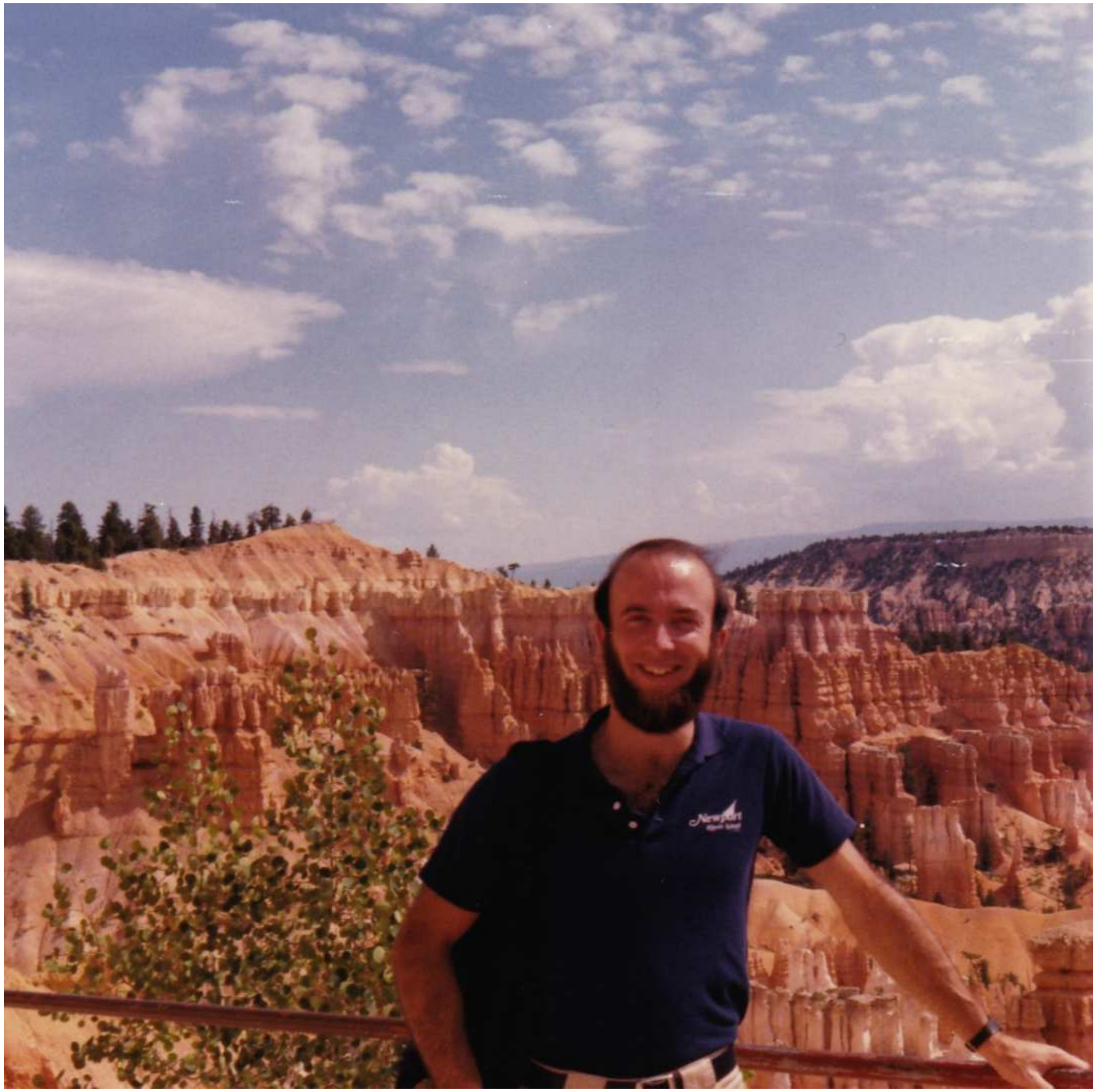
Conference held on the occasion of Luc Vinet's 60th birthday

2014 August 25



Bryce Canyon, 1985

Collaboration

ED-Luc Vinet joint Papers

1. 1984, *Supersymmetry of the Pauli Equation in the Presence of a Magnetic Monopole*, PLB, (130+ citations counted on Google Scholar);
2. 1984, *Dynamical Supersymmetry of the Magnetic Monopole and the $1/r^2$ Potential*, CMP;
3. 1984, *Superspace Formulation of the Dynamic Symmetries of the Dirac Magnetic Monopole*;
4. 1984, *Classical Solutions To Topologically Massive Yang-mills Theory*, Annals of Physics;
5. 1985, *Spectrum (Super)symmetries of Particles in a Coulomb Potential*, Nuclear Physics B;
6. 1985, *Constants Of Motion For A Spin 1/2 Particle In The Field Of A Dyon*, PRL;
7. 1986, *Hidden Symmetries and Accidental Degeneracy for a Spin 1/2 Particle and a Dyon*, LMP;
8. 1987, (A. Kostelecky) *Spectrum Generating Superalgebras*;
9. 1991, (R. Floreanini) *q oscillator realizations of metaplectic representation of quantum osp(3,2)*.

Main Themes

- supersymmetry and superalgebras in quantum mechanical systems
- extended objects, such as monopoles and dyons
- integrable systems

Present Talk

Exact M-theory solutions, branes, and superalgebras

Main themes

- superalgebras of solutions to M-theory and 11-dim supergravity
- extended objects are now intersecting branes
- exact solutions via integrable systems

Based upon

- ED, John Estes, Michael Gutperle, and Darya Krym, arXiv:0806.0605,
  Exact Half-BPS Flux Solutions in M-theory I: Local Solutions
- ED, John Estes, Michael Gutperle, Darya Krym, and Paul Sorba, arXiv:0810.1484,
  Half-BPS supergravity solutions and superalgebras
- John Estes, Roman Feldman, and Darya Krym, arXiv:1209.1845,
  Exact Half-BPS Flux Solutions in M-theory with $D(2,1;c',0)^2$ symmetry: Local Solutions
- Costas Bachas, ED, John Estes, and Darya Krym, arXiv:1312.5477,
  M-theory Solutions invariant under $D(2,1;\gamma) \oplus D(2,1;\gamma)$

M-theory Synopsis

- M-theory unifies superstring theories and their dualities;

- Well-understood in two different limits
  - low-energy approximation $\longrightarrow$ 11-dim supergravity;
  - compactified on a circle $\longrightarrow$ 10-dim Type IIA superstring theory.

- Fundamental constituents
  - M2-branes = 2+1-dim extended objects which carry "magnetic flux"
  - M5-branes = 5+1-dim extended objects which carry "electric flux"

  $\approx$ classical supersymmetric solutions of 11-dimensional supergravity

Geometry of Parallel Brane Solutions

- A stack of N coincident Mp branes has geometry $\mathbb{R}^{p,1} \ltimes \mathbb{R}^{10-p}$ for $p = 2, 5$;
  In coordinates $x^\mu \in \mathbb{R}^{p,1}$ and $\mathbf{y} \in \mathbb{R}^{10-p}$. the metrics are given by,

$$\text{M2} \qquad ds^2 = \left(1 + \frac{c_2 N}{y^6}\right)^{-\frac{2}{3}} dx^\mu dx_\mu + \left(1 + \frac{c_2 N}{y^6}\right)^{\frac{1}{3}} d\mathbf{y}^2$$

$$\text{M5} \qquad ds^2 = \left(1 + \frac{c_5 N}{y^3}\right)^{-\frac{1}{3}} dx^\mu dx_\mu + \left(1 + \frac{c_5 N}{y^3}\right)^{\frac{2}{3}} d\mathbf{y}^2$$

  Preserve half of the maximal number of 32 supersymmetries.

- Space-time geometry near the branes ("Maldacena's near-horizon limit")
  - M2-brane $AdS_4 \times S^7$ $\qquad S^{d+1} = SO(d+2)/SO(d+1)$
  - M5-brane $AdS_7 \times S^4$ $\qquad AdS_{d+1} = SO(d,2)/SO(d,1)$

- Gauge-gravity duality conjectures equivalence with conformal field theories
  - $AdS_4 \times S^7 \quad \Leftrightarrow \quad$ 2+1-dim CFT with 32 supercharges (ABJM)
  - $AdS_7 \times S^4 \quad \Leftrightarrow \quad$ 5+1-dim CFT with 32 supercharges (????)

Geometry of Intersecting Brane Solutions

- For generic collections of branes supersymmetry is completely broken.
- Reduced supersymmetry is preserved for branes at special angles
- Brane configurations with half the supersymmetry (so-called "half-BPS")

branes	0	1	2	3	4	5	6	7	8	9	$\natural$
M2	x	x	x								
M5	x	x		x	x	x	x				
M5'	x	x						x	x	x	x

- Supergravity solutions are NOT known;
- We shall construct their near-horizon limit.

Symmetries

- Bosonic symmetries may be read off from brane configuration

branes	0	1	2	3	4	5	6	7	8	9	♮
M2	x	x	x								
M5	x	x		x	x	x	x				
M5'	x	x						x	x	x	x

– M2-brane $ISO(1,2) \oplus SO(8)$
  - near horizon $SO(2,3) \oplus SO(8)$ = isometry of $AdS_4 \times S^7$
  - Lie superalgebra with 32 supercharges $OSp(8|4,\mathbb{R})$

– M5-brane $ISO(1,5) \oplus SO(4)$
  - near horizon $SO(2,6) \oplus SO(4)$ = isometry of $AdS_7 \times S^4$
  - Lie superalgebra with 32 supercharges $OSp(8^*|4)$

– Full intersection $ISO(1,1) \oplus SO(4) \oplus SO(4)$
  - near horizon $SO(2,2) \oplus SO(4) \oplus SO(4)$ = isometry of $AdS_3 \times S^3 \times S^3$
  - Lie superalgebra with 16 supercharges NOT unique $D(2,1;\gamma) \oplus D(2,1;\gamma)$
  - γ is the ratio of the number of M5 by redM5' branes.

11-dim supergravity

- Uniquely determined by its maximal number of 32 local supersymmetries
  (Cremmer, Julia, Scherk, 1978)

- Field contents
  - Metric $ds^2 = g_{mn} dx^m dx^n$ with $m, n = 0, 1, \cdots, 9, \natural$
  - Majorana gravitino ψ_m
  - Real 4-form field strength $F = \frac{1}{24} F_{mnpq} dx^m \wedge dx^n \wedge dx^p \wedge dx^q$

- Bianchi identity $dF = 0$ and field equations (for vanishing gravitino field),

$$\begin{aligned} d \star F &= \frac{1}{2} F \wedge F \\ R_{mn} &= \frac{1}{12} F_{mpqr} F_n{}^{pqr} + \frac{1}{144} g_{mn} F_{pqrs} F^{pqrs} \end{aligned}$$

Supersymmetric solutions

- Supersymmetry variation of gravitino field (at vanishing gravitino field),

$$\delta\psi_m = D_m\varepsilon + \frac{1}{288}\Big(\Gamma_m{}^{npqr} - 8\delta_m{}^n\Gamma^{pqr}\Big)F_{npqr}\,\varepsilon$$

  – A configuration is supersymmetric if $\delta\psi_m = 0$ has solutions with $\varepsilon \neq 0$.
  – A configuration is half-BPS if $\delta\psi_m = 0$ has 16 linearly independent solutions.

- Conditions $\delta\psi_m = 0$ are reminiscent of Lax equations in an integrable system.
  – with "enough" supersymmetries, they imply field and Bianchi identities, in which case they really are like Lax equations.

- With 16 residual supersymmetries, field and Bianchi equations are implied,
  – and, morally speaking, $\delta\psi_m = 0$ should be equivalent to Lax equations.

Geometry of near horizon intersecting branes

- The $SO(2,2) \oplus SO(4) \oplus SO(4)$ symmetry of the half-BPS intersecting brane configurations dictate the space-time structure to be

$$\left(AdS_3 \times S_2^3 \times S_3^3\right) \ltimes \Sigma$$

  – Σ is a 2-dimensional surface with boundary
  – the product $\ltimes$ is *warped* over Σ

- The metric and 4-form field strength (with $dF = 0$) are restricted by symmetry,

$$\begin{aligned} ds^2 &= f_1^2\, ds^2_{AdS_3} + f_2^2\, ds^2_{S_2^3} + f_3^2\, ds^2_{S_3^3} + ds^2_\Sigma \\ F &= db_1 \wedge \omega_{AdS_3} + db_2 \wedge \omega_{S_2^3} + db_3 \wedge \omega_{S_3^3} \end{aligned}$$

  - $ds^2_{AdS_3}$ is the uniform metric of unit radius, ω_{AdS_3} its volume form;
  - $ds^2_{S^3}$ is the uniform metric with unit radius, ω_{S^3} its volume form;
  - The functions f_1, f_2, f_3 and b_1, b_2, b_3 and the metric ds^2_Σ depend on Σ.

Half-BPS solutions

- Half-BPS solutions are governed by
  - a real function h on Σ,
  - a complex function G on Σ,
  - three real constants c_1, c_2, c_3, satisfying $c_1 + c_2 + c_3 = 0$

- Solutions for the metric coefficients, (similar for 4-form field strength)

$$f_1^6 = \frac{h^2 W_+ W_-}{c_1^6 (G\bar{G} - 1)^2} \qquad f_2^6 = \frac{h^2 W_- (G\bar{G} - 1)}{c_2^3 c_3^3 W_+^2}$$

$$ds_\Sigma^6 = \frac{|\partial h|^6 W_+ W_- (G\bar{G} - 1)}{c_2^3 c_3^3 h^4} \qquad f_3^6 = \frac{h^2 W_+ (G\bar{G} - 1)}{c_2^3 c_3^3 W_-^2}$$

where,

$$W_\pm = |G \pm i|^2 + \gamma^{\pm 1}(G\bar{G} - 1) \qquad \gamma = \frac{c_2}{c_3}$$

Differential equations and algebraic constraints

- Differential equations in local complex coordinates $w, \bar{w}$ on Σ,

$$\partial_w \partial_{\bar{w}} h = 0 \qquad\qquad h \partial_w G = \frac{1}{2}(G + \bar{G}) \partial_w h$$

- Algebraic constraints

$$h > 0 \qquad \gamma(G\bar{G} - 1) > 0 \qquad \text{in interior of } \Sigma$$

$$h = 0 \qquad G = \pm i \qquad \text{on boundary } \partial\Sigma$$

  - invariant under conformal reparametrizations of w
  - independent of $|\gamma|$.

- The differential equations are *solvable*
  - pick a positive harmonic function first;
  - then the equation for G is linear upon superposition with real coefficients;

- But the constraint on G is non-linear.

Associated Integrable System

- Solve algebraic constraint on G by parametrizing $G = e^{(\psi+i\theta)/2}$

on boundary $\partial\Sigma$	$\psi = 0,$	$\theta = \pm\pi$
in interior of Σ	$\gamma\psi > 0$	

- Differential equation in G is non-linear,

$$\partial_w \psi + i\partial_w \theta = \left(1 + e^{-i\theta} \right) \partial_w \ln h$$

- Eliminating ψ between the above equation and its complex conjugate,

$$2\partial_{\bar w}\partial_w \theta + 2\sin\theta \left(\partial_{\bar w}\partial_w \ln h\right) + e^{+i\theta}\partial_w \theta \left(\partial_{\bar w} \ln h\right) + e^{-i\theta}\partial_{\bar w}\theta \left(\partial_w \ln h\right) = 0$$

  – For given h, the above equation is of the Liouville/sine-Gordon type;
  the first equation is its associated Bäcklund transformation system.

  – We have an integrable system in the customary sense;
  as far as we know, this integrable system is new.

The role of $|\gamma|$ and $D(2,1;\gamma)$

- $D(2,1;\gamma)$ is the Lie superalgebra that depends on a continuous parameter γ
  - Its maximal bosonic subalgebra is $SO(2,1)_1 \oplus SO(3)_2 \oplus SO(3)_3$
    generators $T_i^{(a)}$ with $a = 1,2,3$ labeling the algebra
    and $i = 1,2,3$ labeling generators within each algebra
  - Its fermionic generators F transform as $\mathbf{2}_1 \otimes \mathbf{2}_2 \otimes \mathbf{2}_3$
  - Remaining non-trivial structure relation

$$\{F, F\} = c_1 T^{(1)} + c_2 T^{(2)} + c_3 T^{(3)}$$

  with $c_1 + c_2 + c_3 = 0$ and $\gamma = \frac{c_2}{c_3}$

- $D(2,1;\gamma) \oplus D(2,1;\gamma)$ is isometry algebra of solutions;
- $|\gamma|$ is the ratio of the numbers of M5 and M5' branes.
- $\gamma = 0$ corresponds to decompactification of an S^3 and Wigner-Inonu contraction.

Families of exact solutions

We obtain regular, fully back-reacted, near-horizon limits of M2 and M5 branes.

- $\gamma > 0$ gives domain wall or "Janus deformations" of $AdS_4 \times S^7$
- $\gamma < 0$ gives deformations of $AdS_7 \times S^4$
  - M2 ending on or intersecting M5 branes produces conformal defects.
  - equivalently, insertion of surface operators in 6-dimensional (2,0) theory.
  - explicit exact solutions available; setting $h = \mathrm{Im}(w)$,

$$iG = 1 + \sum_{j=1}^{2n} (-)^j \frac{w - \xi_j}{|w - \xi_j|} \qquad \qquad \xi_1 < \xi_2 < \cdots < \xi_{2n}$$

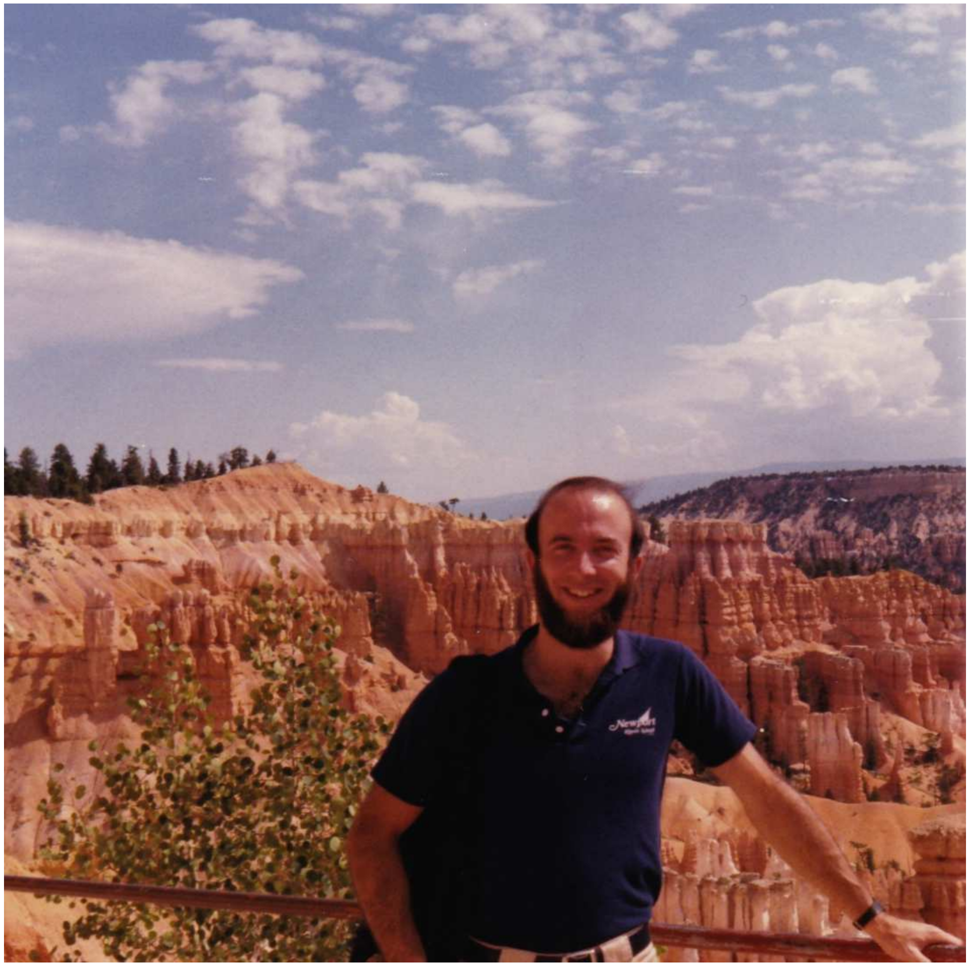
  - represents self-dual strings in arbitrary reps of $SU(N)$

Summary and Open Issues

- Produced explicit map between BPS solutions and 2-d integrable system;
  - Construction of general solution to integrable system is open question;
  - precise holographic dual CFT to be explored.

- Stronger results available for the analogous question in chiral supergravities,
  - 10-d Type IIB (ED, John Estes, Michael Gutperle, arXiv:0705.0022)
  - 6-d Type 4b (Marco Chiodaroli, ED, Yu Guo, Michael Gutperle, arXiv:1107.1722)
  - Chirality provides reduction to genuine Cauchy-Riemann equations;
  - solutions in terms of meromorphic functions on Σ
  - solutions provide CFT duals to Gaiotto-Witten T_N theories (Benjamin Assel, Costas Bachas, John Estes)

HAPPY BIRTHDAY



Bryce Canyon, 1985