Exact half-BPS Solutions to Type IIB supergravity

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- Exact half-BPS Type IIB interface solutions I,
  Local solutions and supersymmetric Janus, arXiv:0705.0022
- Exact half-BPS Type IIB interface solutions II,
  Flux solutions and multi-Janus, arXiv:0705.0024
- Gravity duals of half-BPS Wilson loops, arXiv:0705.1004

Geometry of the solutions

- Construct solutions with 16 supersymmetries to Type IIB supergravity on the following spaces,
  - $AdS_4 \times S^2 \times S^2 \times_w \Sigma$ with $SO(2,3) \times SO(3) \times SO(3)$ symmetry
  - $AdS_2 \times S^4 \times S^2 \times_w \Sigma$ with $SO(2,1) \times SO(5) \times SO(3)$ symmetry

  products warped over 2-dim parameter space Σ (to be specified)
- General *local* solution: exactly in terms of two harmonic functions on Σ
  General *global* solution: all non-singular Type IIB solutions are obtained for geometries whose boundary is locally asymptotically $\sim AdS_5 \times S^5$
- These solutions have varying dilaton, and non-zero 3-form fields.

Motivation

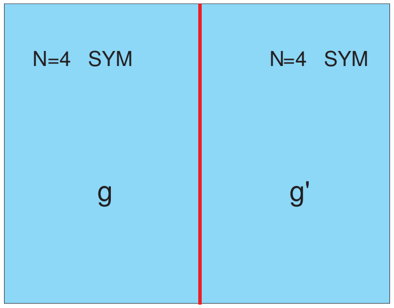
- Construction of AdS dual to half-BPS states in $\mathcal{N} = 4$ SYM
- Closely related to Lin, Lunin and Maldacena (LLM)
  - Half-BPS states on $\mathbf{R} \times S^3$, s-wave, $SU(4)$-highest weight
  - equivalent to free fermion quantum mechanics (Berenstein)
  - LLM provide AdS duals to these states,
  - obtain all half-BPS solutions with $\mathbf{R} \times SO(4) \times SO(4)$ symmetry, with constant dilaton, and vanishing 3-form fields
- $AdS_4 \times S^2 \times S^2 \times_w \Sigma$ geometry: $\mathcal{N} = 4$ SYM with a planar interface
- $AdS_2 \times S^4 \times S^2 \times_w \Sigma$ geometry: $\mathcal{N} = 4$ SYM with a Wilson line

Interface AdS/CFT

• AdS_4 slicings of AdS_5 have 2+1-dim planar interface CFT duals

$$ds^2 = f(\mu)^2(d\mu^2 + ds^2_{AdS_4}) + ds^2_{S^5} \qquad\qquad \phi(\mu)$$

• $AdS_5 \times S^5$ has $f(\mu) = (\cos\mu)^{-1}$, with $\mu \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ and ϕ constant. • More generally, ϕ may vary: Janus solution (Bak, Gutperle, Hirano)



• $SO(2,3)$ isometry group of AdS_4 = conformal group of planar interface

CFT side : susy in the presence of an interface

- Let x^π be a coordinate transverse to a planar interface at $x^\pi = 0$
- Bulk Lagrangians on each side are supersymmetric $\delta\mathcal{L}_\pm = \partial_\mu X^\mu_\pm$

$$\begin{aligned}\mathcal{L} &= \theta(x^\pi)\mathcal{L}_+ + \theta(-x^\pi)\mathcal{L}_- \\ \delta\mathcal{L} &= \partial_\mu\Big(\theta(x^\pi)X^\mu_+ + \theta(-x^\pi)X^\mu_-\Big) - \delta(x^\pi)\left(X^\pi_+ - X^\pi_-\right)\end{aligned}$$

- Can one compensate for $X^\pi_+ - X^\pi_-$ by interface Lagrangian $\mathcal{L}_I$?
- Assume coupling varies across interface,

$$\mathcal{L} = \frac{1}{g(x^\pi)^2}\mathcal{L}_{\{\mathcal{N}=4\}} + \mathcal{L}_I$$

- Bulk susy transformations must also be modified on interface

Allowed interface supersymmetries

Susy driven by $\psi^t Y \psi + \mathrm{cc}$ term in $\mathcal{L}_I$, where Y acts on $SU(4)$ indices

$$Y \to \mathrm{diag}[d_1, d_2, d_3, d_4] \qquad\qquad d_i \text{ real } \geq 0$$

- 0 supersymmetries $\mathrm{diag}[0\ 0\ 0\ 0]$, global $SU(4)$ R-symmetry,
  - CFT dual to Janus solution of Bak, Gutperle, Hirano
- 4 supersymmetries $\mathrm{diag}[1\ 0\ 0\ 0]$, global $SU(3)$ R-symmetry;
  - CFT: Clark, Freedman, Karch, Schnabl; AdS dual D'Hoker, Estes, Gutperle
- 8 supersymmetries $\mathrm{diag}[1\ 1\ 0\ 0]$, global $SO(2) \times SO(3)$ R-symmetry;
- 16 supersymmetries $\mathrm{diag}[1\ 1\ 1\ 1]$, global $SO(3) \times SO(3)$ R-symmetry;
  - AdS dual : This talk !

AdS side : Type IIB Supergravity

- The fields of Type IIB sugra are

g_{MN}	metric	$M,N=0,1,\cdots,9$	
B	axion/dilaton	$P \sim dB$	(contains χ, ϕ)
$B_{(2)}$	antisymm	$G_{(3)} \sim (dB_{(2)} - BdB^*_{(2)})$	
$C_{(4)}$	antisymm	$F_{(5)} = dC_{(4)} + \mathrm{Im}(\bar B_{(2)} dB_{(2)})/8$	$F_{(5)} = *F_{(5)}$
ψ_M	gravitino	Weyl spinor	
λ	dilatino	Weyl spinor	

- The susy variation equations for the spinors are (J.H. Schwarz, 1983)

$$\delta\lambda = iP\cdot\Gamma\,\mathcal{B}^{-1}\varepsilon^* - \frac{i}{4}(G_{(3)}\cdot\Gamma)\varepsilon$$

$$\delta\psi_M = D_M\varepsilon + \frac{i}{4}(F_{(5)}\cdot\Gamma)\,\Gamma_M\varepsilon - \frac{1}{16}\Big(\Gamma_M(G_{(3)}\cdot\Gamma) + 2(G_{(3)}\cdot\Gamma)\Gamma_M\Big)\mathcal{B}^{-1}\varepsilon^*$$

AdS dual to Interface with 16 susys

- Isometry group must be $SO(2,3) \times SO(3) \times SO(3)$;
- Space-time is $AdS_4 \times S_1^2 \times S_2^2$ warped over a 2-dim parameter space Σ

$$e^{i_1} = f_1 \hat{e}^{i_1} \qquad ds^2 = f_1^2 ds^2_{S_1^2} + f_2^2 ds^2_{S_2^2} + f_4^2 ds^2_{AdS_4^2} + ds^2_\Sigma$$

$$e^{i_2} = f_2 \hat{e}^{i_2} \qquad G_{(3)} = g_a e^{45a} + i h_a e^{67a}$$

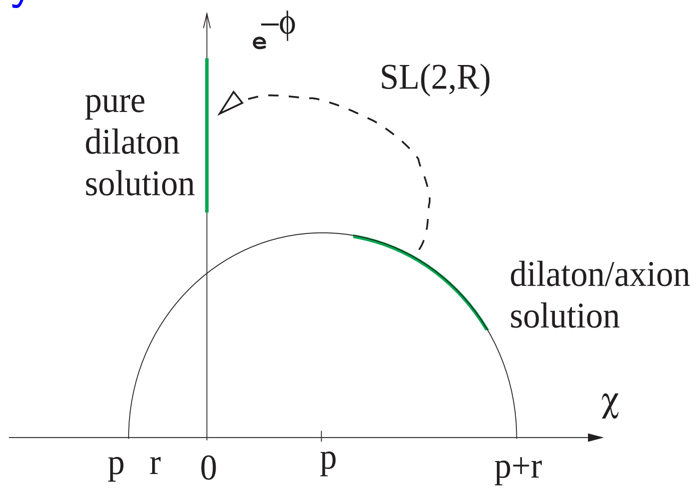
$$e^m = f_4 \hat{e}^m \qquad F_{(5)} = f_a \left(-e^{0123a} + \varepsilon^a{}_b \, e^{4567b} \right)$$

- index ranges: $m = 0,1,2,3;\quad i_1 = 4,5;\quad i_2 = 6,7;\ a = 8,9;\quad \varepsilon^{89} = 1$
- orthonormal frames $\hat{e}^m, \hat{e}^{i_1}, \hat{e}^{i_2}$ resp. on AdS_4, S_1^2, S_2^2 with unit radius
- e^a is the orthonormal frame on Σ with $ds^2_\Sigma = e^a \otimes e^a$
- functions f_1, f_2, f_4, f_a are real, g_a, h_a, B complex

- Reduce BPS equations $\delta\lambda = \delta\psi = 0$ to the above Ansatz
- Method of Killing spinors/vectors
  (Gauntlett, Martelli, Pakis, Waldram – Pilch, Warner)
  - well-suited for problems with $G_{(3)} = 0$
  - less suited for varying dilaton and $G_{(3)} \neq 0$
- $AdS_4 \times S^2 \times S^2$ BPS eqs set up by Gomis and Rommelsberger
  $AdS_2 \times S^4 \times S^2$ BPS eqs set up by Lunin
  - identified one harmonic function,
  - but obtaining $f_1, f_2, f_4, \cdots$ still requires solving differential equations
  - which they did not succeed in doing.
- We work directly with the BPS equations;
  - we shall use Killing spinors as well,
  - but we shall not follow the standard methods of GMPW – PW
  - this will allow us to find the general solution, exactly.

General simplifications

- Solutions with 16 susys
  - axion/dilaton runs over a geodesic path
  - by $SL(2,\mathbf{R})$ symmetry of Type IIB, every solution to the BPS equations may be mapped to a solution with vanishing axion field, and real g_a, h_a, B.
  - Every solution to the BPS equations automatically solves the Bianchi and field equations



The role of Killing spinors

- The dilatino BPS equation, for non-constant dilaton, $\partial_M \phi \neq 0$,

$$0 = 4(\partial_M \phi)\Gamma^M \mathcal{B}^{-1} \varepsilon^* - (G_{(3)} \cdot \Gamma)\varepsilon$$

  – allows for at most 16 independent solutions ε

- The gravitino BPS equation should impose no further restrictions on ε,

$$0 = D_M \varepsilon + \frac{i}{4}(F_{(5)} \cdot \Gamma)\, \Gamma_M \varepsilon - \frac{1}{16}\Big(\Gamma_M (G_{(3)} \cdot \Gamma) + 2(G_{(3)} \cdot \Gamma)\Gamma_M\Big)\mathcal{B}^{-1}\varepsilon^*$$

- On any one of the maximally symmetric components, AdS_4, S_1^2, S_2^2
  – ε should reduce to a "Killing spinor" (see e.g. Gomis and Rommelsberger)
  $=$ spinor of maximal rank

Killing spinors

- Consider spheres $S^d = SO(d+1)/SO(d)$ of even dimension d.
- Maurer-Cartan connection ω, in spinor representation, $V \in SO(d+1)$,

$$\omega = V^{-1} dV = \frac{1}{4} \omega_{\bar{m}\bar{n}} \gamma^{\bar{m}\bar{n}} \qquad \bar{m}, \bar{n} = 1, \cdots, d+1$$

  – decomposes into a $SO(d)$ connection ω_{mn}, with $m, n = 1, \cdots, d$ and an orthonormal frame $e_m = \omega_{m(d+1)}$ on S^d

- The parallel transport equation $(d + \omega)\chi = 0$ is solved by $\chi = V \chi_0$,
  – No constraints on $\chi_0 \Rightarrow$ solution space always has *maximal rank*
  – Coincides with the Killing spinor equation on S^d,

$$\left(d + \frac{1}{4} \omega_{mn} \gamma^{mn} - \frac{1}{2} \eta \, e_m \, \gamma^m \, \gamma^{d+1} \right) \chi_\eta = 0 \qquad \eta = \pm 1$$

- Analogously for $AdS_d = SO(2, d-1)/SO(1, d-1)$ spaces.

Reducing the BPS equations

- Use Killing spinors on $AdS_4 \times S^2 \times S^2$ as basis for the susy parameter ε,

$$\varepsilon = \sum_{\eta_1,\eta_2,\eta_3} \chi^{\eta_1,\eta_2,\eta_3} \otimes \zeta_{\eta_1,\eta_2,\eta_3}$$

  - $\chi^{\eta_1,\eta_2,\eta_3}$ fixed basis, $\eta = \pm$ independently;
  - $\zeta_{\eta_1,\eta_2,\eta_3}$ are 2-component spinors on Σ,

  $\Rightarrow$ 2 complex algebraic reduced dilatino eqs
  $\Rightarrow$ 6 complex algebraic reduced gravitino eqs along $AdS_4 \times S^2 \times S^2$;
  $\Rightarrow$ 4 complex differential reduced gravitino eqs.

- Symmetries of reduced BPS eqs lead to only non-vanishing components,

$$\begin{array}{ccccc} \zeta_{+\pm\pm-} & \sim & \zeta^*_{+\pm\mp+} & \sim & \alpha \\ \zeta_{-\pm\mp-} & \sim & \zeta^*_{-\pm\pm+} & \sim & \beta \end{array}$$

Reducing the BPS equations cont'd

- Introduce local complex coordinates $w, \bar w$ on Σ, $ds^2_\Sigma = 4\rho^2 |dw|^2$
- The algebraic equations may be used to solve for $f_1, f_2, f_4, g_a, h_a, f_a$,

$$f_1 = \alpha\bar\beta + \bar\alpha\beta \qquad g_z + ih_z = -4\alpha(\rho\beta)^{-1}\partial_w\phi$$

$$f_2 = i\bar\alpha\beta - i\alpha\bar\beta \qquad g_z - ih_z = +4\beta(\rho\alpha)^{-1}\partial_w\phi$$

$$f_4 = \alpha\bar\alpha + \beta\bar\beta \qquad f_z = \frac{i\nu}{2\alpha\beta} - \frac{\alpha^4 - \beta^4}{4\rho\alpha^2\beta^2}\partial_w\phi$$

- Then, eliminate these functions from the differential BPS equations,
- Two of the differential BPS eqs are equivalent to Cauchy-Riemann eqs,

$$\partial_w(\rho\alpha^2) + \rho\beta^2\partial_w\phi = 0 \qquad \Rightarrow \qquad \alpha^2 + \beta^2 = \bar\kappa\, e^{-\bar\lambda}\rho^{-1}e^{-\phi}$$

$$\partial_w(\rho\beta^2) + \rho\alpha^2\partial_w\phi = 0 \qquad \Rightarrow \qquad \alpha^2 - \beta^2 = \bar\kappa\, e^{+\bar\lambda}\rho^{-1}e^{+\phi}$$

  – κ, λ arbitrary holomorphic, respectively 1-form and 0-form.

Mapping to a new integrable system

- Eliminating α, β from the remaining 2 differential eqs
  - leaves 2 complex differential eqs for ϕ, ρ in terms of κ, λ.
  - Drastic simplification by changing variables to $i\mu \equiv \lambda - \bar\lambda$, and

$$\frac{\mathrm{sh}(2\phi + 2\lambda)}{\mathrm{sh}(2\phi + 2\bar\lambda)} = e^{2i\vartheta} \qquad \rho^8 = \hat\rho^8 (\sin 2\mu)^2 \frac{\kappa^4 \bar\kappa^4}{16} \frac{\sin\vartheta + \sin\mu}{(\sin\vartheta - \sin\mu)^3}$$

- Two differential eqs for $\vartheta, \hat\rho$ in terms of κ, μ become,

$$\begin{aligned} \partial_w \vartheta - (e^{-i\vartheta} + i \sin\mu)(\cos\mu)^{-1} \partial_w \mu &= -i\kappa \hat\rho^2 e^{i\vartheta/2} \\ \partial_w \vartheta - 2 e^{-i\vartheta} (\cos\mu)^{-1} \partial_w \mu &= -2i \partial_w \ln \hat\rho^2 \end{aligned}$$

$=$ System of Bäcklund transfs for the partial differential eq,

$$\partial_{\bar w} \Big(\partial_w \vartheta - 2(\cos\mu)^{-1} (\partial_w \mu) e^{-i\vartheta} \Big) + \mathrm{c.c.} = 0$$

- This system is automatically integrable.

General Exact solution

- A final change of variables, $\psi \equiv \hat{\rho}^{-2}(\cos\mu)e^{-i\vartheta/2}$ maps to a linear system,

$$\begin{aligned} \partial_w \psi &= -\kappa \cos\mu \\ \partial_w \bar{\psi} &= (i\psi - \bar{\psi} \sin\mu)(\partial_w \mu)(\cos\mu)^{-1} \end{aligned}$$

- whose general solution is given via κ, μ, or equivalently harmonic h_1, h_2,

$$\psi = i h_1 e^{-\bar{\lambda}} + h_2 e^{+\bar{\lambda}} \qquad e^{2\lambda} = i\partial_w h_1 / \partial_w h_2 \qquad \kappa^2 = 4i\partial_w h_1 \partial_w h_2$$

- All fields of the Ansatz may be expressed in terms of h_1, h_2, e.g.

$$e^{4\phi} = \frac{2h_1 h_2 |\partial_w h_2|^2 - h_2^2 W}{2h_1 h_2 |\partial_w h_1|^2 - h_1^2 W} \qquad W \equiv \partial_w h_1 \partial_{\bar{w}} h_2 + \text{c.c.}$$

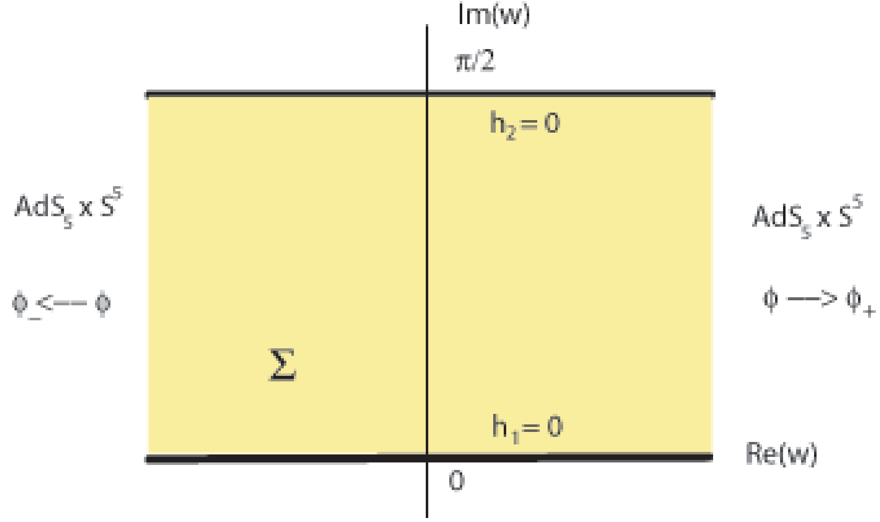
$$\rho^8 = \frac{W^2}{h_1^3 h_2^3} \Big(2h_1 |\partial_w h_2|^2 - h_2 W \Big) \Big(2h_2 |\partial_w h_1|^2 - h_1 W \Big)$$

$AdS_5 \times S^5$ and Janus with 16 susys

- We readily obtain a 2-parameter family of non-singular solutions,

$$h_1 = \mathrm{Im}\left(e^{w-\phi_+} - e^{-w-\phi_-}\right) \qquad h_2 = \mathrm{Re}\left(e^{w+\phi_+} + e^{-w+\phi_-}\right)$$

- For $\phi_+ = \phi_-$ gives $AdS_5 \times S^5$
- For $\phi_+ \neq \phi_-$, dilaton varies
  $=$ Janus solution with 16 susys
  – in $\partial\Sigma$: $h_1 h_2 = 0$
  – in Σ : $W \leq 0$, $h_1, h_2 \geq 0$



General regularity conditions

- More generally, regularity will require that inside Σ, we have

$$0 < e^{4\phi} = \frac{2h_1 h_2 |\partial_w h_2|^2 - h_2^2 W}{2h_1 h_2 |\partial_w h_1|^2 - h_1^2 W} \qquad W \equiv \partial_w h_1 \partial_{\bar w} h_2 + \text{c.c.}$$

$$0 \leq \rho^8 = \frac{W^2}{h_1^4 h_2^4} \Big(2h_1 h_2 |\partial_w h_2|^2 - h_2^2 W \Big) \Big(2h_1 h_2 |\partial_w h_1|^2 - h_1^2 W \Big)$$

- Set of manifestly sufficient conditions inside Σ,

$$h_1 > 0 \qquad h_2 > 0 \qquad W \leq 0$$

  – These are obeyed by $AdS_5 \times S^5$ and Janus with 16 susys

- Still need boundary conditions on $\partial\Sigma$.

General regularity conditions cont'd

- General solution manifold $\mathcal{S}$ is specified by 3 conformal data: Σ, h_1, h_2.
  - Assume boundary of $\mathcal{S}$ is locally asymptotic to $AdS_5 \times S^5$;
  - Asymptotic $AdS_5 \times S^5$ regions originate from isolated points on $\partial\Sigma$;

  $\Rightarrow$ Away from those points, $\partial\Sigma$ produces *interior points* of $\mathcal{S}$;

  $\Rightarrow$ Either S_1^2 or S_2^2 must shrink to zero on $\partial\Sigma$ (but never AdS_4)

  $\Rightarrow$ Either $f_1 = 0$ or $f_2 = 0$ on $\partial\Sigma$ (but f_4 is never zero);
- The form of the solution then imposes boundary conditions on h_1, h_2,

$$f_1^2 f_4^2 = 4e^{+2\phi}\, h_1^2 \qquad\qquad f_1 = 0 \quad \Rightarrow \quad (h_1 = 0 \quad \& \quad \partial_w h_2 = 0)$$

$$f_2^2 f_4^2 = 4e^{-2\phi}\, h_2^2 \qquad\qquad f_2 = 0 \quad \Rightarrow \quad (h_2 = 0 \quad \& \quad \partial_w h_1 = 0)$$

- Equivalent to two coupled electro-statics problems with
  - alternating Neumann and vanishing Dirichlet conditions on $\partial\Sigma$
  - $h_1, h_2 > 0$ in the interior of Σ

General regular solutions

- Map the domain Σ onto the lower half-plane with complex coordinate u.
  - The boundary $\partial\Sigma$ is then the real axis $\mathbf{R}$.
  - Points e_i on $\partial\Sigma$ where Dirichlet $\leftrightarrow$ Neumann, $i = 1, 2, \cdots, 2g+2$.
- Construction of h_1, h_2 via hyperelliptic curve of genus g, defined by

$$s(u)^2 = (u - e_1)(u - e_2)\cdots(u - e_{2g+1}) \qquad e_{2g+1} < \cdots < e_1 < e_0 = \infty$$

- The meromorphic differentials $\partial h_1, \partial h_2$ may be written down explicitly,

$$\partial h_1 = -i\frac{P_1(u)du}{s(u)^3} \qquad \partial h_2 = -\frac{P_2(u)du}{s(u)^3}$$

  - for two real polynomials P_1, P_2 of degree $3g+1$,
  - Neumann and Dirichlet conditions automatically satisfied,
  - behavior at branch points $du/(u-e_i)^{3/2}$ guarantees asymptotic $AdS_5 \times S^5$

General regular solutions cont'd

- Regularity requires that
  - P_1, P_2 have g common complex zeros u_a, $a = 1, \cdots, g$,
  - P_1 has $g+1$ real zeros α_b, $b = 1, \cdots, g+1$,
  - P_2 has $g+1$ real zeros β_b, $b = 1, \cdots, g+1$, satisfying the ordering

$$\alpha_{g+1} < e_{2g+1} < \beta_{g+1} < e_{2g} < \cdots < e_2 < \alpha_1 < e_1 < \beta_1$$

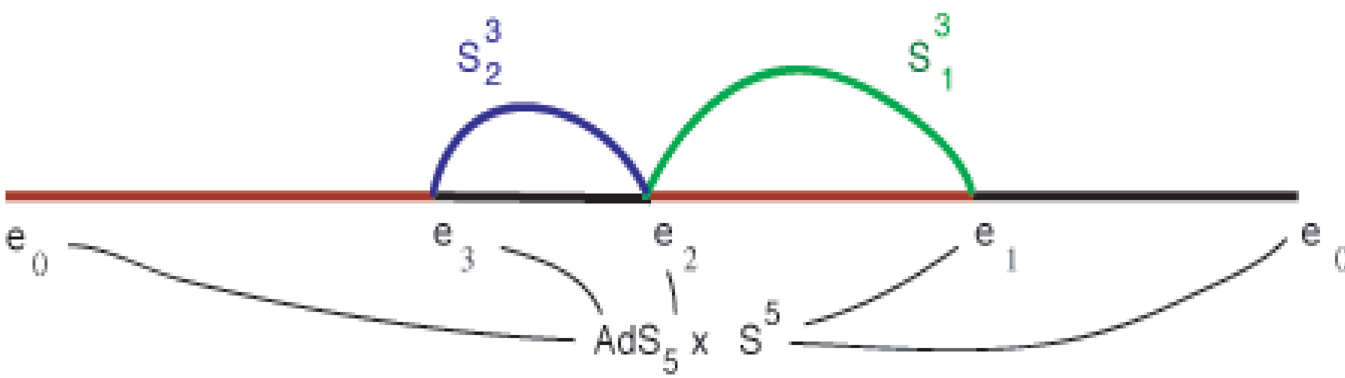
- It only remains to ensure that the Dirichlet conditions VANISH,

$$\mathrm{Im} \int_{e_{2j}}^{e_{2j-1}} \partial h_1 = \mathrm{Im} \int_{e_{2j+1}}^{e_{2j}} \partial h_2 = 0 \qquad\qquad j = 1, \cdots, g$$

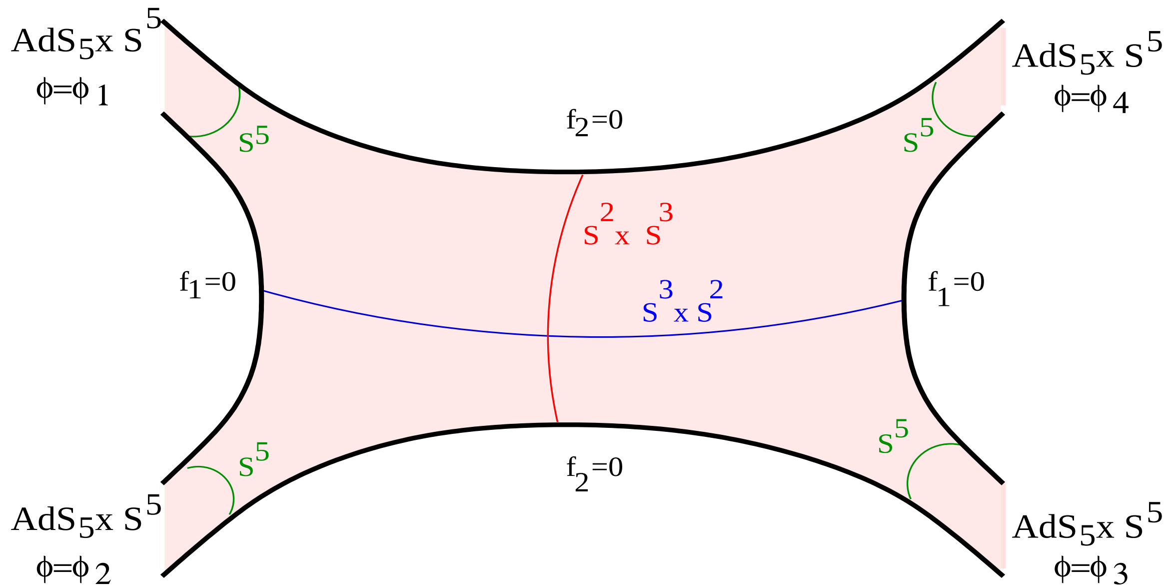
- Given the branch points e_i and the ordered real zeros α_b, β_b,
  - The above period relations determine the g complex zeros u_a.
  - The geometry of the allowed moduli space is known explicitly for $g = 1$,
  - and is known locally for $g \geq 2$; analogous to instanton moduli space.

Topology of regular solutions

- $2g+2$ branch points = different asymptotic boundary $AdS_5 \times S^5$ regions
  - each with its independent constant dilaton limit
- There are g independent pairs of homology 3-spheres, $j = 1, \cdots, g$
  - $S^3_{1j} = [e_{2j}, e_{2j-1}] \times_f S^2_1$ NSNS 3-form charges $\mathcal{H}_j = \int_{S^3_{1j}} H_{(3)}$
  - $S^3_{2j} = [e_{2j+1}, e_{2j}] \times_f S^2_2$ RR 3-form charges $\mathcal{F}_j = \int_{S^3_{2j}} F_{(3)}$
  - e.g. genus 1

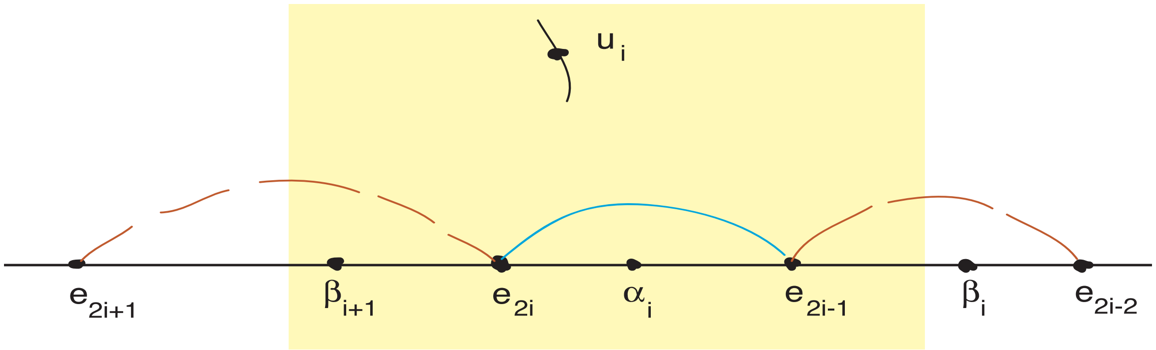


e.g. Genus 1 solution for AdS_4



- For any genus g, solutions have $2g + 2$ asymptotic $AdS_5 \times S^5$
- Number of free parameters of solution is $4g + 6$,
  – including: restoring the axion by $SL(2, \mathbf{R})$, and overall radius

Topology change : a collapsing branch cut



$$\partial h_1 = \frac{(u-u_i)(u-\bar{u}_i)(u-\alpha_i)}{(u-e_{2i})^{3/2}(u-e_{2i-1})^{3/2}}(\partial h_1)_{g-1} \qquad \partial h_2 = \frac{(u-u_i)(u-\bar{u}_i)(u-\beta_{i+1})}{(u-e_{2i})^{3/2}(u-e_{2i-1})^{3/2}}(\partial h_2)_{g-1}$$

- As $e_{2i-1} \to e_{2i}$ we must have $\alpha_i \to e_{2i}$, and $\mathrm{Im} \int \partial h_1 = 0$ forces $u_i \to e_{2i}$
- Two possibilities

  (A) $\beta_{i+1} \to e_{2i}$ gives topology change $(\partial h_{1,2})_g \to (\partial h_{1,2})_{g-1}$

  (B) $\beta_{i+1} \not\to e_{2i}$ gives $\partial h_1 \to (\partial h_1)_{g-1}$ but leaves a singular ∂h_2

  $\sim$ the probe limit: a D5 (or NS5) brane remains

Total branch cut collapse

- Collapse of all branch cuts produces probe brane limit,
  - m_R D5 branes and m_{NS} NS5 branes with $m_R + m_{NS} = g$
  - leads to a simple explicit solution,

$$h_1 = -2i(w - \bar{w}) \left(1 + \frac{C_0}{|w|^2}\right) + \sum_{j=1}^{m_R} \frac{C_j}{\ell_j} \ln \left| \frac{w + i\ell_j}{w - i\ell_j} \right|^2$$

$$h_2 = -2(w + \bar{w}) \left(1 + \frac{D_0}{|w|^2}\right) - \sum_{i=1}^{m_{NS}} \frac{D_i}{k_i} \ln \left| \frac{w + k_i}{w - k_i} \right|^2$$

  - for real $k_i, \ell_j, C_j, D_i, C_0, D_0$, and positive
  - RR and NSNS 3-form charges given by

$$\mathcal{F}_j = C_j/\ell_j \qquad\qquad \mathcal{H}_i = D_i/k_i$$

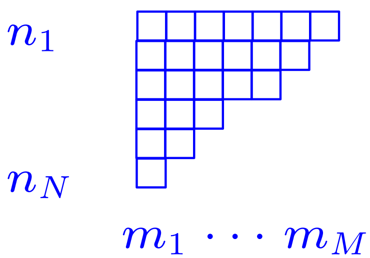
CFT dual to AdS_4 solutions (in progress)

- The AdS_4 factor indicates the presence of an interface.
- For $g = 0$, CFT dual has interface operators (built from bulk fields).
- For $g \geq 1$, several gauge groups
  - different species of $\mathcal{N}=4$, decoupled away from interface
  - interact only via the interface
  - are coupled via extra massless fields on the interface
  - On AdS side, extra massless fields arise from S^3 shrinking to zero
- For $g \geq 1$, as branch cuts collapse,
  - and we approach the limit with probe branes,
  - recover massless string excitations from probe D5 branes

    of De Wolfe, Freedman, Ooguri – Skenderis, Taylor
- Our solutions are fully back-reacted geometries with D5 and NS5 branes

The Wilson loop with 16 supersymmetries

- On CFT side, half-BPS Wilson loop in $\mathcal{N}=4$, $SU(N)$ SYM,



$$\mathrm{Tr}_R \exp i \int dt \, (A_0 + n^I \phi_I) \qquad |n| = 1$$

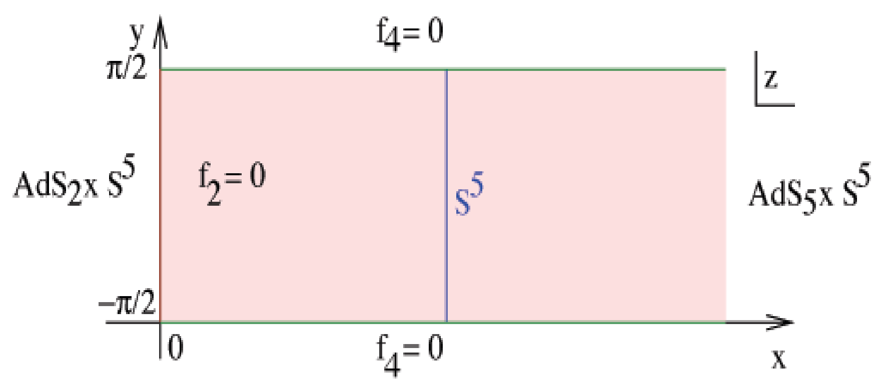
  - Invariant under $SO(2,1) \times SO(3) \times SO(5)$
- On AdS side, quantum numbers can be realized via probe branes
  - fundamental rep $\square$ via D3 probe (Maldacena – Rey, Yee)
  - symmetrizations of $\square$ via D5 probes (Drukker, Fiol)
- General representation R, (Gomis, Passerini)
  - probe D5 branes $j = 1, \cdots, M$, with m_j units of F1 dissolved

  OR probe D3 branes $i = 1, \cdots, N$, with n_i units of F1 dissolved

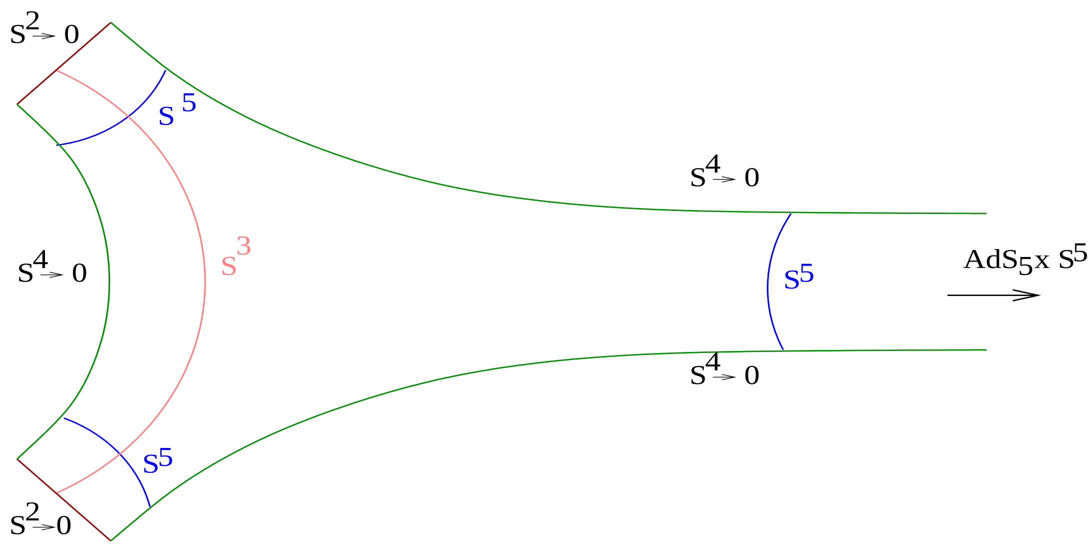
Wilson loop AdS dual supergravity solution

- CFT symmetry $SO(2,1) \times SO(3) \times SO(5)$ requires
- AdS dual geometry $AdS_2 \times S^2 \times S^4 \times_w \Sigma$
  - Formally, obtained by analytic continuation from $AdS_4 \times S^2 \times S^2 \times_w \Sigma$
  - Analytic continuation may send regular to singular, & change # susys

  $\Rightarrow$ Topologies and # moduli different
- Our methods: general solution via harmonic functions h_1, h_2 on Σ.



e.g. The genus 1 solution for AdS_2



- For any genus g, solutions have only a single asymptotic $AdS_5 \times S^5$
- There are g independent homology S^3 carrying RR 3-form charges
- Number of free parameters of solution is $2g + 5$ for $g \geq 1$ (3 for $g = 0$).

Geometries with CFT duals and 16 susys (in progress)

- Can one construct all solutions with 16 susys to Type IIB sugra ?
- Can one construct all solutions with 16 susys which have a CFT dual ?
  - View as AdS duals to deformations of $\mathcal{N} = 4$ SYM
  - Expect a subgroup H of $SU(2,2|4)$ with 16 susys to be preserved
  - Semi-simple H first, with maximal bosonic subgroup H_B

H	H_B	space-time	sol's
$SU(2\|2)\times SU(2\|2)$	$SO(4)\times SO(4)\times R$	$M_4\times S^3\times S^3$	LLM
$OSp(4\|4^*)$	$SO(2,3)\times SO(3)\times SO(3)$	$AdS_4\times S^2\times S^2\times\Sigma$	DEG
		$AdS_4\times S^3\times\Sigma$	?
$OSp(4^*\|4)$	$SO(2,1)\times SO(3)\times SO(5)$	$AdS_2\times S^2\times S^4\times\Sigma$	DEG
$SU(2\|4)$	$SO(3)\times SO(5)$	$M_3\times S^2\times S^4$	?
$SU(1,1\|4)$	$SO(2,1)\times SO(5)$	$AdS_2\times S^5\times E_3$	?
$SU(2,2\|2)$	$SO(2,4)\times SO(3)$	$AdS_5\times S^2\times E_3$	?

Further open problems

- Half-BPS solutions to Type IIB supergravity are surprisingly manageable;
- Regular solutions to AdS_2 and AdS_4 problems with other topologies ?
- Can one derive a reduced quantization of only the half-BPS states
  – Free fermion/matrix reduction may be derived directly from LLM
    (Maoz, Rychkov – Grant, Maoz, Marsano, Papadodimas, Rychkov)
- Unified approach to 16 susy solutions from subgroups of $SU(2,2|4)$?

The End

Interface operators

- Using the same requirements as for the interface transformations,

$$\mathcal{L}_\psi = \frac{\partial_\pi g}{g^3} \mathrm{tr} \left(y_1 \bar\psi \gamma^\pi \psi + \frac{i}{4} y_2^{ij} \bar\psi \gamma^\pi \rho^{ij} \psi - \frac{i}{2} y_3^{ijk} \psi^t \rho^{ijk} \psi + cc \right)$$

$$\mathcal{L}_\phi = \frac{\partial_\pi g}{2g^3} \mathrm{tr} \Big(z_1^{ij} \partial_\pi (\phi^i \phi^j) + 2 z_2^{ij} \phi^{[i} D_\pi \phi^{j]} - i z_3^{ijk} \phi^i [\phi^j, \phi^k] \Big)$$

$$\mathcal{L}_{\phi^2} = \frac{(\partial_\pi g)^2}{2g^4} \mathrm{tr} \Big(z_4^{ij} \phi^i \phi^j \Big)$$

- The interface terms have the following $SU(4)_R$ representations,

$$\begin{cases} y_1 & \mathbf{1} \\ z_1,\ z_4 & \mathbf{1} \oplus \mathbf{20'} \end{cases} \qquad \begin{cases} y_2,\ z_2 & \mathbf{15} \\ y_3,\ z_3 & \mathbf{10} \oplus \mathbf{10}^* \end{cases}$$

- The **10**, $\mathbf{10}^*$, **15**, **20'**, couple to sugra fields; **1** couples to strings.

The maximally supersymmetric iCFT

- $D = \mathrm{diag}[1\ 1\ 1\ 1]$, 8 Poincaré (16 conf) susy: global $SO(3) \times SO(3)$;

$$
\begin{aligned}
\mathcal{L}_I &= \frac{\partial_\pi g}{2g^3} \mathrm{tr}\left(i\psi^t \mathcal{C} \psi + i\psi^\dagger \mathcal{C} \psi^* - 4i\phi^2[\phi^4, \phi^6] \right) \\
&\quad + \left(g^{-3}(\partial_\pi g)\partial_\pi - 2g^{-4}(\partial_\pi g)^2 \right) \mathrm{tr}\left((\phi^2)^2 + (\phi^4)^2 + (\phi^6)^2 \right)
\end{aligned}
$$

- Last term may be absorbed into kinetic term for ϕ^i

  by rescaling $\phi^i \to g^2 \phi^i$ for $i = 2, 4, 6$ and $\phi^i \to \phi^i$ for $i = 1, 3, 5$

$$
\mathcal{L}_I^{\mathrm{rescaled}} = \frac{\partial_\pi g}{2g^3} \mathrm{tr}\Big(i\psi^t \mathcal{C}\psi + i\psi^\dagger \mathcal{C}\psi^* - 4ig^6 \phi^2 [\phi^4, \phi^6] \Big)
$$

- The rescaled theory admits a conformal limit where g is a step function.