Development of an Inexpensive Toroidal Charge Detector

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This summer’s work concerned the research and development of a relatively inexpensive charge detector. The detector, which is toroidal in shape, measures the amount of charge that passes through its center by allowing this pulse of charge to electromagnetically induce an equal amount of charge on the detector itself; it is designed in the context of being used at the end of an accelerator beam line. It operates on a similar principle to that of a commercial integrating current transformer device, but is considerably cheaper. The new detector designed and built this summer was able to achieve a better sensitivity in measuring low charges than the commercial device by optimizing the behavior of the ferromagnetic materials used in the core of the detector. The new detector’s lowest measurable charge is 0.013 pC as opposed to the 0.1 pC limit of the commercial detector. Its output, which takes the form of undamped voltage oscillations, was also made easier to analyze over previous models, in that its quality factor was increased from 6 to 27 so that small signals persist considerably and can then be analyzed to determine their charge.

I. INTRODUCTION

Many commercial charge detectors exist and are used in the context of modern elementary particle physics and accelerator beam physics. They work over large dynamic ranges from those that can sense single electrons, like photomultiplier tubes, to those that measure millions or even billions of elementary charges, like Faraday cups.

This summer’s work focused on developing a charge detector similar to the ICT, or Integrating Current Transformer, operating on a similar principle but produced for a mere fraction of the price (the ICT currently costs $ 10,000). Both the new detectors and the ICT are designed to sit at the end of an accelerator beam. Bunches of charge from the beam then pass through the center of the detectors (both designs are toroidal), electromagnetically inducing an equal charge to accumulate within the detector. In the ICT, the charge in the detector is measured by discharging it and then integrating the current from the discharge. In the new detectors, this charge is measured by directly monitoring voltage within the device.

These toroidal detectors are advantageous because they operate on a Stokes’s law argument (leading directly from Faraday’s law of electromagnetic induction), allowing them to detect charge over an area simply by instrumenting only its bounding perimeter. This creates a light detector that offers complete area coverage.

They do not interfere with the charge signals they measure; beams pass through them unimpeded. This is in contrast to a Faraday cup, for example, which destroys the signal that it measures. Having a nondestructive means of charge detection is useful in experiments that themselves absorb the charge signal, but in which the charge of a beam pulse needs to be measured before entering the experiment itself. This was the case in a calorimeter experiment performed at SLAC National Accelerator Laboratory, in which both the ICT and new detector were tested and used.

The goal of this summer’s work was to characterize the performance of the new devices and to improve their sensitivity, so that they may measure lower charges than the commercial ICT. It is hoped that these detectors, being relatively inexpensive, will be useful in educational contexts, such as for use on small student accelerators.

This paper will explore the theory behind the operation of the new charge detectors, the construction of the devices and the effects of the materials chosen, and the methods by which the devices were tested, calibrated, and used. We will then explain the numerical techniques used to analyze the output of the device. The performance of the devices will then be evaluated and dynamic ranges of charge that each detector can measure will be presented. Finally, opportunities for further work will be discussed.

II. THEORY

The charge detectors are designed to rest at the end of a beam line. The device, which is toroidal in shape (and will thus often be referred to simply as the toroid), measures the bunch charge from the beam that passes through its center. The core of the toroid comprises a ring of ferrite, which is covered along three of its surfaces in copper sheeting. Around the circumference of the toroid, capacitors are attached, forming closed loops with the edges of the copper sheeting. This setup constitutes an arrangement of capacitances in parallel.

The toroid also has resistive and inductive elements, however, due to the magnetic properties of its ferrite core. Ferrite, a ferromagnetic material, has a relative permeability $\mu$ that is both complex and a function of frequency, $f$ [5]:

$$\mu(f) = \mu'(f) - i\mu''(f)$$  \hspace{1cm} (1)

The real part of permeability, $\mu'$, represents the ability of a material to store magnetic energy, whereas the
imaginary part, $\mu''$, characterizes the degree of energy loss associated with its magnetic field. This imaginary component can then be considered a resistance, just as the real component of $\mu$ provides inductance to the device. The ferrite core and capacitors thus form all the requisite elements of an LRC circuit.

As will be shown, the relative ratio of inductance to resistance, which is determined by the ratio of real to imaginary permeability, plays an important role in numerous quantities describing the behavior of the toroids output. When this ratio is high, the performance of the device is optimal.

*FIG. 1: Dependence of real and complex parts of magnetic permeability of ferrite with frequency [1]. At low frequencies, the real part of $\mu$ dominates; at intermediate frequencies, both components are of similar magnitude; at high frequencies, both drop significantly, but the imaginary part is much greater than the real part. This plot corresponds to Material 61 from Fair-Rite, which was used as the core of the high-frequency toroid.*

The exact ratio $\mu'$ to $\mu''$ depends both on frequency and on the exact chemical constitution of the ferrite used; a sample plot of both the real and imaginary parts of permeability versus frequency is shown in Figure 1. At low and intermediate frequencies (for the ferrite considered, up to about 12 MHz), the inductance of the device is prominent; at high frequencies, the inductance is negligible. The resistive properties of the toroid become significant at intermediate frequencies (about 10 MHz) and remain so at very high frequencies. Thus, for intermediate frequencies, the toroid behaves as an LRC circuit; at high frequencies, it is simply RC.

When a beam bunch $Q_{\text{beam}}$ passes through the center of the device, it creates a time-varying current, directed parallel to the toroid’s axis. The magnetic field of this current is thus azimuthal to the axis of the toroid, directed along its circumference, and varying in time.

To understand how this beam charge affects the device itself, consider its cross-sectional area, which is defined by the closed loop created by the capacitor and copper sheeting. The juxtaposition of this cross-section and the beam current pulse is shown in Figure 2.

*FIG. 2: Diagram of cross-sectional cut through the toroid detector. The beam pulse, which passes through the center of the toroid, is in blue. The thicker black line represents the copper sheeting, which envelopes three sides of the ferrite. Capacitors (thin black line and circuit symbol) connect the sheeting at certain points along the toroid to form a closed loop around the toroid’s cross-section; it is around this loop that charge flows.*

As the magnetic field lines from the beam pulse are perpendicular to this surface and are of time-varying magnitude, according to Faraday’s law, an electromotive force (EMF) is induced around the toroids cross-section from this time-varying magnetic flux, $\Phi$:

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

This EMF causes current to flow around this closed loop, charging the capacitor to a final value of $Q_{\text{toroid}}$. As discussed, the different structural components of the toroid act as circuit elements connected in series in this charging process. If certain impedance conditions are met [4],

$$Z_L >> Z_C, R$$

then the final charge induced on the toroid in this charging phase will equal the charge that passed through the device initially:

$$Q_{\text{beam}} = Q_{\text{toroid}}$$
The derivation of these impedance conditions will be given in II.1. The higher the ratio $\mu'/\mu''$, the better impedance condition in Equation 3 is satisfied.

This gives a simple, nonintrusive method for inferring the beam charge as long as the charge on the toroids capacitors may be measured. If the peak voltage across the toroid can be measured, charge may be found using the definition of capacitance:

$$C = \frac{Q}{V}$$ \hspace{1cm} (5)

II.1. Charging and Discharging Phases

Both the charging and discharging phases of the toroid can be described using Kirchhoff’s loop rule of circuit analysis, in which an expression for the electric potential drops around a loop is equated to the electromotive force supplied around the loop. The loop is, in this case, that defining the toroid’s cross-section, created by the copper sheeting and capacitor. In the charging phase, the electromotive force is induced by the changing magnetic flux supplied by the beam current. In the discharging phase, there is no electromotive force around the loop.

The loop may be described as a circuit using its capacitance, $C$, the resistance resulting from the ferrite’s imaginary permeability, $R$, the ferrite’s self inductance, $L$, and the mutual inductance between the toroid and the beam, $M$. This mutual inductance for the toroidal geometry happens to equal the toroid’s self inductance $L$.

$$L = M. \hspace{1cm} (6)$$

This will be shown in VII.

The forms of the toroid’s charging and discharging equations will now be explored. A sample of both phases is plotted in Figure 3 on page 4.

II.1.1. Charging the Toroid

The voltage drops contributed by the different LRC circuit elements of the toroid must sum to equal the electromotive force induced by the beam current, $\mathcal{E}$:

$$V_R + V_L + V_C = \mathcal{E} \hspace{1cm} (7)$$

Writing the current flowing around the loop of the toroid’s cross-section as $I(t)$, this becomes [4]:

$$RI(t) + L \frac{dI(t)}{dt} + \frac{1}{C} \int_{-\infty}^{t} I(\tau)d\tau = \mathcal{E} \hspace{1cm} (8)$$

Using Faraday’s law and the definition of mutual inductance, the electromotive force induced by the beam current $I_{beam}(t)$ is:

$$\mathcal{E} = M \frac{dI_{beam}(t)}{dt} \hspace{1cm} (9)$$

The current induced to flow around the cross-section of the toroid is then given by the differential equation [4]:

$$RI(t) + L \frac{dI(t)}{dt} + \frac{1}{C} \int_{-\infty}^{t} I(\tau)d\tau = M \frac{dI_{beam}(t)}{dt} \hspace{1cm} (10)$$

If this equation were to be Fourier transformed with respect to time, it would give the following relationship between the beam current and the current flowing around the toroid in the frequency domain [4]:

$$RI(\omega) + i\omega LI(\omega) + \frac{1}{i\omega C} I(\omega) = i\omega MI_{beam}(\omega) \hspace{1cm} (11)$$

Or, rearranging:

$$I(\omega) = \frac{i\omega M}{R + i\omega L - \frac{1}{\omega C}} I_{beam}(\omega) \hspace{1cm} (12)$$

The terms in the denominator may be recognized as expressions for resistive, inductive, and capacitive impedance, respectively, or $R$, $Z_L$, and $Z_C$. If the inductive impedance were to dominate over the resistive and capacitive terms, as given by the condition in Equation 3, then, using Equation 6, we find:

$$I(\omega) \approx \frac{M}{L} I_{beam}(\omega) = I_{beam}(\omega) \hspace{1cm} (13)$$

Returning to the time domain, we can see that the current flowing around the toroid is the same as that passing through its center, so, upon integrating both sides of the equation over time we can see that the charge built up on the toroid equals the beam charge passing through the toroid, recovering our initial assumption about the toroid’s operation.

$$Q_{toroid} = Q_{beam} \hspace{1cm} (15)$$

The charge on the toroid will then decay away in amplitude in its discharging phase, so $Q_{beam}$ corresponds to the largest magnitude of charge on the toroid during the time vicinity of the entire beam pulse event.

The validity of the assumption that $Z_L >> Z_C$, $R$ can be examined by considering the nature of $I_{beam}(\omega)$. As a beam pulse is nearly an impulse or delta function in nature, $I_{beam}(\omega)$ contains very high frequency components. Thus, while at high frequencies $\mu''$ (the resistive component of the ferrite’s behavior) is greater than $\mu'$ (the inductive component), the magnitude of the toroid’s inductive impedance, $|Z_L| = \omega L$, is still much greater than the resistive impedance $R$ due to this extra multiplicative factor of $\omega$. The capacitive impedance $|Z_C| = \frac{1}{\omega C}$ becomes negligible at high frequencies.
II.1.2. Discharging the Toroid

Once the beam current \( I_{beam}(t) \) has settled to a constant zero value after the beam pulse, the toroid’s capacitors have charged up to their final value of \( Q_{toroid} \), which under the chosen material’s impedance conditions equals \( Q_{beam} \). The toroid then spontaneously discharges. There is no EMF around the loop of its cross-section, so Kirchhoff’s law reads:

\[
RI(t) + L\frac{dI(t)}{dt} + \frac{1}{C} \int_{-\infty}^{t} I(\tau) d\tau = 0 \quad (16)
\]

where \( I(t) \) gives the rate of discharge of the toroid’s capacitors. In terms of the charge on the toroid, \( Q \), this becomes a homogeneous linear second order differential equation \([4]\):

\[
R\frac{dQ(t)}{dt} + L\frac{d^2Q(t)}{dt^2} + \frac{Q(t)}{C} = 0 \quad (17)
\]

So long as the following condition is met, the solution \( Q(t) \) to the above equation will be underdamped \([4]\), oscillating with some characteristic frequency \( \omega_0 \) with an amplitude decaying with time:

\[
\frac{R}{2} \sqrt{\frac{C}{L}} < 1 \quad (18)
\]

The unforced response of the toroid discharges according to \([4]\):

\[
Q(t) = [A \cos(\omega_0 t) + B \sin(\omega_0 t)]e^{-\frac{R}{2L}t} \quad (19)
\]

\[
\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \quad (20)
\]

where \( Q(0) = Q_{toroid} \).

The condition for underdamping will be met if the characteristic ringing frequency \( \omega_0 \) is in the intermediate frequency range of the ferrite, such that the ferrite’s inductive properties are more prominent than its resistive properties. This can be achieved by choosing \( \omega_0 \) to be in this frequency range by adjusting the values of the capacitors on the toroid.

The toroids have been designed so that they are underdamped in the discharging phase, with the ringing frequency ranging from the order of 100 kHz to the order of 10 MHz.

In addition to setting the characteristic frequency of the toroid output, the choices of capacitance and ferrite (and thus \( C, R, \) and \( L \)) also set the rate of exponential decay of the amplitude of the charge oscillations on the capacitor. Thus, the toroid output \( Q(t) \) has a characteristic value for its quality factor given by \([1]\):

\[
QF = \sqrt{\frac{4L}{CR^2} - 1} \quad (21)
\]

The greater the ratio of inductance to resistance, the higher the quality factor of the toroid’s output.

\[\text{FIG. 3: Plot of the charging and discharging of the toroidal charge detector. The charging phase is shown in blue and the discharging phase in red. The peak voltage of 68 mV corresponds to the maximum charge built up on the capacitor, } Q_{toroid}, \text{ which equals the charge that passed through the detector, } Q_{beam}.\]

II.2. Material Differences

The two models of toroid, the low-frequency toroid and the high-frequency toroid, are distinguished by the ferrite in their cores. The low-frequency toroid has a core made from Material J from Magnetics Inc., whose permeability versus frequency plot is shown in Figure 4. The high frequency toroid uses Material 61 from Fair-Rite, whose permeability plot features in Figure 1.

As already discussed, the type of ferrite determines the frequency dependences of both \( L \) and \( R \) of the toroid (from its ratio of \( \mu' \) to \( \mu'' \)), thus setting the toroid’s characteristic frequency \( \omega_0 \) and quality factor. The ringing frequency also determines the minimum noise level of the toroid’s output, as the intrinsic noise from the amplifier is given roughly by \([6]\):

\[
\text{Noise} = \left(0.95 \frac{nV}{\sqrt{Hz}}\right) \sqrt{f} \quad (22)
\]

A comparison of the ringing frequencies, quality factors, and relative noise levels of the two different toroids with these distinct cores is given in Table I.
FIG. 4: Dependence of real part of magnetic permeability of ferrite with frequency for J Material, used as the core of the low frequency toroid [2].

TABLE I: Comparison of ferrite materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>Material J</th>
<th>Material 61</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ringing Frequency (MHz)</td>
<td>0.16</td>
<td>11.5</td>
</tr>
<tr>
<td>Quality Factor</td>
<td>6</td>
<td>27</td>
</tr>
<tr>
<td>Relative Noise Level</td>
<td>1</td>
<td>8.5</td>
</tr>
</tbody>
</table>

II.3. Comparison with ICT

The toroids measure $Q_{toroid}$ by directly monitoring the voltage across the capacitor throughout its charge and discharge, then inferring the peak voltage, which corresponds to $Q_{toroid}$. The commercial ICT device operates on a slightly different principle, instead measuring the current as the capacitor is discharged through a more complicated external circuit, which is shown in Figure 5. This external circuit causes it to discharge quickly, always taking about 0.1 µs. It does not ring as the toroids do; a sample of its output is shown in Figure 6. The current is then recorded through an ammeter and this signal is then integrated to find the charge built up on the capacitor from the beam pulse.

III. METHODS

III.1. Detector Construction

The low-frequency toroid was built in Spring 2015 by Peihao Sun. This summer, the high-frequency toroid was built by Tyler Chesebro and the author. Two ferrite rings of Material 61 were used as the core of the toroid. The dimensions of the cores are given in Table II.

A ring and sheet of copper sheeting were fashioned to go around the outer circumference and bottom of the core. A segment of copper pipe of the same radius as the inner radius of the ferrite core was used as the inner copper layer. These distinct copper parts were all brazed together to form a toroidal shell for the ferrite; the result can be seen in Figure 7.

Once the copper shell was complete, the ferrite cores were slotted into it, and capacitors were brazed onto the copper (one wire to the outer copper ring, one to the inner in order to form a closed loop), completing the detector's construction. The finished toroid is shown in Figure 8.

The values of capacitance were chosen such that the ringing frequency of the toroid (expression given in Equation 20) corresponded to the peak value of the ferrite’s real permeability, $\mu'$, at which the relative ratio of inductance to resistance is maximized and at which the output

FIG. 5: A circuit diagram of the circuit within the ICT [3]. Where the toroids just have an inductor and resistor, the ICT has a transformer connecting it to a parallel LRC circuit. This is responsible for its different method of discharge.

FIG. 6: Sample output of the Bergoz ICT. Voltage data may be converted to current and then integrated with respect to time to calculate the charge measured by the ICT.
TABLE II: Ferrite core dimensions.

| Inner radius (cm) | 4.0 |
| Outer radius (cm) | 5.2 |
| Height (cm)       | 1.3 |

FIG. 7: The three-sided toroidal copper shell in which the ferrite core rests, made of pieces of copper sheeting and pipe that have been brazed together.

has the highest quality factor. This capacitance may be estimated for this chosen frequency $f_{\text{ideal}}$ by assuming that, at this frequency, the resistance of the toroid is negligible:

$$C = \frac{1}{4\pi^2 f_{\text{ideal}}^2} \frac{1}{L(f_{\text{ideal}})} \quad (23)$$

Substituting an expression for $L(f_{\text{ideal}})$, which has been evaluated using the dimensions of the ferrite core, this becomes:

$$C = \frac{1}{4\pi^2 f_{\text{ideal}}^2} \frac{1}{\mu_r(f_{\text{ideal}}) \times 1.34 \ \text{nH}} \quad (24)$$

The capacitance was altered until this optimal value was found, corresponding to $C = 1 \ \text{nF}$.

III.1.1. Calibration of the Toroid

However, due to the limitations of the discrete values of capacitance available for use, in order to obtain a 1 nF toroid, the capacitance had to be asymmetrically distributed about the toroid’s circumference. This leads to some equilibration of voltages between the uneven capacitors around the toroid during the charging and discharging of the toroid. Effectively, this decreases the charge on the toroid that is measured from the ideal value of $Q_{\text{beam}}$ by a constant value. This value was found by two methods of calibration, the first described in III.2, and the second by comparison of toroid charge measurements with ICT charge measurements for the same beam pulse passing through each device. These methods agreed, giving a calibration constant for the high-frequency toroid that:

$$Q_{\text{beam}} = 2.7Q_{\text{toroid}} \quad (25)$$

The low-frequency toroid has its capacitance symmetrically distributed around it circumference, so it does not require a calibration factor; simply, $Q_{\text{beam}} = Q_{\text{toroid}}$.

III.2. Testing

By sending a pulse of known charge through the toroid, the output of the toroid may be compared to the known charge in order to test its performance. Due to limited access to a beam line, the known charge was typically provided by a pulse generator, which created a current pulse of a specified amplitude and width in time. To obtain charges below the dynamic range of the pulse generator, attenuators were used to modulate the amplitude of the current signal.

This current pulse was sent through coaxial cable, which split to send a positive current pulse through the center of the toroid, and then rejoined with itself into...
another coaxial cable that terminated into a 50 Ω oscilloscope. The signal measured on the oscilloscope could then be integrated with respect to time to determine the magnitude of the charge pulse, simply using Ohm’s law:

\[ Q_{\text{input}} = \int I(t) dt = \int \frac{V(t)}{R} dt \]  

(26)

A sample pulse is shown in Figure 9.

\[ \text{FIG. 9: An input current pulse of known charge used to calibrate the toroidal charge detector. The oscilloscope reading of this current pulse terminating into 50 Ω is shown. This voltage versus time data may be converted to current versus time data and integrated to find the input charge, which is reported on the plot.} \]

The output of the toroid is then recorded by monitoring the voltage across one of its capacitors with time by recording the voltage signal into an oscilloscope. Figure 11 depicts the pulsing setup. The peak voltage on the capacitor may be found within the voltage signal, and from this the peak charge on the capacitance of the toroid, which should equal the input charge, can be deduced. Such a sample output is shown in Figure 10; this signal was given by the low-frequency toroid.

\[ \text{FIG. 10: The output of the low-frequency toroid when the current pulse from Figure 9 passed through it. The charge on the beam should equal the peak charge on the toroid, which is calculated from the peak voltage. This charge on the toroid agrees with the known beam pulse charge to within 1%.} \]

\[ \text{FIG. 11: A cartoon schematic of the pulsing setup used to calibrate the toroid. The input current pulse from the pulse generator is in blue; the toroid output is in red. Both the input and output signals terminate at the oscilloscope.} \]

III.3. Use Along Beam Line

When the toroids were used to measure charge at SLAC, they were suspended at the end of the beam line. This allowed the beam bunch charges to pass through the toroid unimpeded as soon as they exited the beam line. Coaxial cables ran the output signal from the toroid’s amplifier to an oscilloscope. A picture of this setup is seen in Figure 13.

\[ \text{FIG. 13: A picture of the setup used to measure charge at SLAC. The toroid is suspended at the end of the beam line, and the output signal is run through coaxial cables to the oscilloscope.} \]

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\[ ^2 \text{A subtlety about this will be discussed in V.1.1} \]
TABLE III: Amplifier properties.

<table>
<thead>
<tr>
<th>Amplifier</th>
<th>Low-Frequency Amp</th>
<th>High-Frequency Amp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order of Frequency of Operation</td>
<td>100 kHz</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Gain</td>
<td>2857</td>
<td>714</td>
</tr>
</tbody>
</table>

FIG. 12: A picture of the amplifier unit used to amplify the toroid’s output signal. Amplifier designed and built by Peter Yu.

III.4. Data Analysis

The specifics of the data analysis techniques will be covered in V.1, but a quick outline is given here. The decaying sinusoidal output from the toroid’s discharge may be fit to a functional form and then extrapolated to infer the peak charge on the toroid. The output may also be cross-correlated with a template output (whose charge is known) to find this charge. Fitting the data works better on larger signals corresponding to higher charges, including those that are saturated beyond the capabilities of the amplifier; correlation has the benefit of working on signals from very low charges.

IV. RESULTS

Using the methods of analysis that will be described in V.1, values for charge measured by the high-frequency toroid were obtained for all events on all runs taken at the experiment at SLAC; the same calculations were made for the ICT. The high-frequency toroid measured charges as high as 54 pC (a value obtained by fitting to saturated output) and as low as 0.013 pC (inferred from cross-correlation). Note that the smallest possible charge to which the output could be fitted to a functional form was 0.050 pC.

A sample output’s data and functional fit are shown in Figure 15; this signal corresponds to a charge of 0.2 pC.

The data for a signal of charge 0.098 pC is also given in Figure 14.

While the low-frequency toroid was not used at SLAC, from testing with the pulse generator, its minimum measurable charge was determined to be 0.5 pC. No estimate has yet been obtained for its maximum charge.
V. DISCUSSION

V.1. Methods of Analysis

The charge on the toroid, \( Q_{toroid} \), which provides the estimate of the beam charge \( Q_{beam} \), can be found from the peak voltage \( V_{peak} \) of the toroid’s output signal:

\[
Q_{toroid} = CV_{peak}. \tag{27}
\]

where \( C \) is the effective capacitance of the toroid (the sum of the individual capacitances, as they are connected in parallel). This voltage \( V_{peak} \) occurs at time \( t_0 \), the exact moment at which the charging phase completes; after \( t_0 \), the toroid is within its discharge phase.

Two different methods may be used to infer the actual peak voltage of the toroid: fitting and cross-correlation.

V.1.1. Fitting the Output

Finding \( V_{peak} \) is often not as simple as finding the maximum voltage value in the output. This is because of an effective low-pass filtering that occurs at the amplification stage of data measurement.

The capacitor plates reach this peak potential difference upon the end of the charge-up phase. This phase occurs quickly, in a matter of tens of microseconds, after having had a constant potential difference of zero. It is similar to a delta function spike. Thus, this initial rise up to \( V_{peak} \) depends on the contributions from very high frequency components, unlike the rest of the toroid’s output, as the discharge phase occurs predominantly at a single, much lower frequency component, the characteristic ringing frequency of the toroid.

The amplifier on the toroid’s output is designed to work at this characteristic frequency; it does not effectively amplify the higher frequency components in the signal. The higher frequency components are thus artificially lowered in amplitude, so they do not contribute as much as they should to the rise in voltage from its constant zero value to \( V_{peak} \). As a result, the measured signal always has a slightly lower recorded charging peak than exists in the true signal.

Since the discharge is not affected by this effective filtering of the higher frequency components, its shape is true, and may be used to infer the peak voltage. The shape of the discharge on its own (the data \( V(t) \) for time \( t > t_0 \)) can be fitted to the functional form expected of the output,

\[
V_{fit}(t) = [A \cos(\omega t) + B \sin(\omega t)]e^{-\alpha t} \tag{28}
\]

The result of the fitting algorithm applied to data gives values for \( A, B, \omega, \) and \( \alpha \), which are found as the optimal parameters that minimize the squared residual between the data \( V(t) \) and the calculated fit values \( V_{fit}(t) \),

\[
res = \sum_i (V_{fit}(t_i) - V(t_i))^2 \tag{29}
\]

Using these optimal parameters, the fit function \( V_{fit}(t) \) may be extrapolated backwards to the time at which the toroid actually reached its peak voltage, \( t_0 \). The value of the fit at this time, \( V_{fit}(t_0) \), gives an estimate of the true peak voltage \( V_{peak} \) that actually existed on the toroid upon the end of its charging phase. A sample of such a fit overlaid on raw data is shown in Figure 16; note that the value of the fit at \( t_0 \) is greater than that of the data at \( t_0 \), as expected and desired.

In order to successfully fit the data, some further signal processing may be necessary if the signal is particularly noisy. In some cases, a moving average was used to smooth the data by effectively filtering out some higher frequency components. This moving or boxcar average converted the raw data \( V(t_i) \) to \( V_{average}(t_i) \) using the following definition:

![High Frequency Toroid Output Signal](image)

![High Frequency Toroid Fitted Output](image)

FIG. 15: Above: output data of high-frequency toroid from SLAC event; data has been smoothed using a moving average filter. Below: fit (red) to output data (blue) of high-frequency toroid from same output as above, corresponding to a charge of 0.2 pC.
FIG. 16: A fit (red) for the raw output (blue) of the low-frequency toroid. The functional fit is found by fitting the functional form of Equation 28 to the raw output data from the toroid’s discharge and is then extrapolated backwards to 0 µs, where the charging of the toroid occurs. The value of the fit at this time gives an estimate for $V_{\text{peak}}$. Note that this estimate is greater than the highest datum from the toroid’s raw output.

$$V_{\text{average}}(t_i) = \frac{1}{\text{width} + 1} \sum_{j=i-\text{width}/2}^{i+\text{width}/2} V(t_j)$$  \hspace{1cm} (30)

Typically, this width was set such that noise components above 100 MHz were filtered out of the data. This made the standard fitting algorithm used generally more efficient at finding the minimum of the residual (given in Equation 29), as the removal of high frequency noise reduced the size of the residual and made the decaying sinusoidal trend (at the characteristic frequency) in the data easier to find.

The difference is also visually noticeable, as can be seen in Figure 17.

The longer the signal persists, the more data the fitting algorithm has with which to work, and thus the easier it is to make a more precise and accurate fit. The higher the quality factor, the more cycles of oscillation a signal goes through before its amplitude decays to the point at which it is negligible compared to the output’s noise level. With a high quality factor, even signals that are never much larger than the noise level may be fitted, as the signal lasts long enough to find the sinusoidal decay above the noise. Thus, a toroid with a higher quality factor is preferable, as a larger range of signals may be fitted.

Fitting is also useful for very high signals that are saturated, such that any true voltages above a certain value $V_{\text{max}}$ are clipped to the value of $V_{\text{max}}$. Eventually, the signal will decay in amplitude to voltages below $V_{\text{max}}$, and will thus regain its decaying shape. If a fit is made to the data for all values after this point, the frequency and decay constant of the output may be found, and this fit may be extrapolated backwards over all of the cycles for which the output was saturated to infer a value for $V_{\text{peak}}$, from the very first saturated cycle.

A sample of saturated output and the fit made to it are given in Figure 18.

The success of the fit may be judged by a number of factors; these are necessary to avoid making accidental fits to noise that has a vaguely similar shape to that expected from the output. The first two are physical - both the characteristic frequency and decay constant of the toroid’s output are set by the material properties of the toroid and have been determined. If the fit parameters do not agree with the known values of these properties, then the fit does not correspond to a signal from the toroid.

FIG. 17: Above: the raw (unaveraged) output of the high-frequency toroid. Below: the averaged output of the high-frequency toroid, obtained when a moving average was applied to the output above.
Consider a template output from the toroid in question, obtained from a good, clear signal from the toroid that corresponds to a known charge. This template has the same characteristic ringing frequency and exponential decay constant as any other output from the same toroid; all that differs is the amplitude of the decaying sinusoids. If the output in question can be compared to the template in some way that gives the ratio of their amplitudes, the charge of said output could be found relative to the charge from the template, which is known.

Cross-correlation provides such a method of comparison. The cross-correlation $H(t_i)$ of a signal $V(t_i)$ with a template $G(t_i)$ is defined as:

$$H(t_i) = \sum_j V(t_j) \times G(t_i+j)$$

This acts to compute an inner product between $V$ and $G$ at some time shift $t_i$ between their arguments. At some certain shift, the signal and template will be aligned such that their charging phases are coincident with one another. As both have the same frequency and decay constant, this aligns every peak in $V$ with every peak in $G$, and the same for the troughs. This maximizes the value of this sliding inner product; this value is proportional to the product of the peak amplitude of $V$, and the peak amplitude of $G$. If $G$ has been normalized such that its autocorrelation (the cross-correlation of $G$ with $G$) has such a maximum value of unity, then the maximum value of the cross-correlation of $V$ with $G$ will simply equal the peak amplitude of $V$. Along with a constant calibration factor to correct for the filtering effect discussed in V.1.1, this provides an estimate of $V_{\text{peak}}$.

An example of this cross-correlation method is shown in Figure 19.

The method of cross-correlation has the benefit that, even if the signal to noise ratio of the output is so low that the decaying sinusoidal signal cannot be seen by eye and that the fitting algorithm cannot find an appropriate functional fit, correlation may still infer the amplitude of the signal, since it averages all frequency components not equal to the characteristic frequency (i.e., the noise) to zero upon integration with a template of this characteristic frequency.

As this allows for the calculation of charge from outputs without visible signals, one must be careful that correlation is not used just on pure noise, which will inevitably carry some component corresponding to the toroid’s characteristic frequency. Thus, before a correlation is performed, a fast Fourier transform of the output is calculated to ensure that the toroid’s characteristic ringing frequency is the dominant frequency component.

The other metric is statistical; if the squared residual between the data and the fit is above a set threshold, or if the fitting algorithm never found a minimum for this residual, then the fit is rejected as not adequately agreeing with the data itself, as the integrity of the fit relative to the actual data is paramount.

V.1.2. Correlating the Output

Another method exists that may be used to infer the peak voltage of the signal; this is cross-correlation. This method allows for very small signals to be analyzed, corresponding to lower charges than measurable by fitting.

3 This factor, which corrects for the fact that the raw output’s recorded peak is smaller than the actual peak voltage on the toroid, was found by comparing estimates for peak voltage from correlation with those from fitting, which already corrects for this filtering effect, for hundreds of toroid outputs.
FIG. 19: Cross-correlation of low-frequency toroid output with a template, for the same output as shown in Figure 10. The template has been normalized such that the maximum value of this correlation equals the peak voltage of the toroid’s output, corresponding to the value to which the toroid was charged up.

in the output. If it is, the correlation is accepted as corresponding to a charge value from a true signal that is simply not visible amidst the noise; if not, the output is rejected as noise not containing a signal.

Correlation may not be used on saturated signals.

Typically, for signals for which both methods are possible, fitting and correlation agree with each other to within 5%.

V.2. Comparison Between Detectors

A comparison between the three detectors is given in Table IV. TL represents the low-frequency toroid, TH the high-frequency toroid.

<table>
<thead>
<tr>
<th>Device</th>
<th>ICT</th>
<th>TL</th>
<th>TH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Detectable Charge (pC)</td>
<td>0.1</td>
<td>0.5</td>
<td>0.013</td>
</tr>
<tr>
<td>Maximum Detectable Charge (pC)</td>
<td>160</td>
<td>n/a</td>
<td>54</td>
</tr>
<tr>
<td>Quality Factor</td>
<td>n/a</td>
<td>6</td>
<td>27</td>
</tr>
<tr>
<td>Cost</td>
<td>$10,000</td>
<td>$100</td>
<td>$800</td>
</tr>
</tbody>
</table>

The high-frequency toroid performs better than the low-frequency toroid, being more sensitive by nearly a factor of fifty. Its output also has a much higher quality factor than that of the low-frequency toroid, which in part explains why it is able to measure lower charges: smaller signals persist for longer in the high-frequency toroid before they decay into the noise, making them easier to fit.

The high-frequency toroid has achieved the goal of making a detector more sensitive to low charges than the ICT, by nearly an order of magnitude of sensitivity, having measured a lowest charge of 0.013 pC to the ICT’s 0.1 pC. The dynamic range of the high-frequency toroid (a factor of about 5000 between lowest and highest measurable charges) is better than that of the ICT (a factor of about 1600). It is also about a factor of ten cheaper than the ICT.

These improvements are largely due to the choice of higher quality ferrite for the toroid’s core and the optimization of the ratio of inductive to resistive impedance (real to imaginary permeability of the ferrite) at the chosen characteristic ringing frequency, set by the choice of capacitance.

VI. CONCLUSIONS

To return to the goals posed in the introduction, the behavior of the toroids has been characterized and their outputs have been calibrated so that the toroid signal $V(t)$ may be analyzed to give a consistently accurate estimate of the charge passing through the device, $Q_{\text{beam}}$, from the charge on the toroid, $Q_{\text{toroid}}$. Two distinct methods were used to infer this charge value from the output, fitting the output to a specific functional form (a sinusoid of exponentially decaying amplitude) and cross-correlating the output with a template signal of known charge.

A new detector, the high-frequency toroid, was constructed using a ferrite core with more desirable properties concerning its complex permeability, which affects the effective inductance and resistance of the toroid in both its charging and discharging phases.

This new toroid performed better than previous attempts at such a detector, with a quality factor of 27 compared with the low-frequency toroid’s quality factor of 6. It was also more sensitive than the ICT, an initial hope for the project. The lowest charge measurable by the ICT is 0.1 pC, or about 620,000 elementary charges, whereas the lowest charge measurable by the high-frequency toroid is 0.013 pC, or about 70,000 elementary charges.

The performance of the high-frequency toroid may be further improved by evenly distributing variable capacitors around its circumference, such that a symmetric distribution of the ideal capacitance may be achieved around the detector. This would remove the need for any calibration factors in analyzing its output and also possibly allow for further optimization of the frequency of operation to a value at which the ferrite has its absolute maximum ratio of inductive behavior to resistive behavior according to its complex permeability.

Since the devices are both cheap and relatively light for what they do, as they detect over an area simply by instrumenting its perimeter, it is hoped that they may
someday also be scaled up in size, perhaps in the context of creating a large toroidal detector to be launched into space to study cosmic rays.

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VII. APPENDIX

Self-inductance of the toroid, \( L \), is defined as the ratio of magnetic flux linked through the area of its cross-section per unit current flowing around the loop defining its cross-section (that made by the copper sheeting and capacitor) \[4\]:

\[
L = \frac{\Phi_B}{I_{loop}} \tag{32}
\]

It describes the extent to which changing current in the device itself creates electromotive forces within the device. To evaluate this, we may assume that the current \( I_{loop} \) flows uniformly around the toroid’s circumference (i.e., with azimuthal symmetry) \[4\]. The magnetic field generated by such a current is directed azimuthally around the toroid’s circumference, with magnitude depending only on radial distance from the toroid’s center (where the beam current is located), \( r \):

\[
B_{toroid}(r) = \frac{\mu_0 I_{loop}}{2\pi r} \tag{33}
\]

To find the magnetic flux passing through the toroid’s cross-section, we must compute:

\[
\Phi_B = \iint_{\text{cross-section}} B \cdot dS \tag{34}
\]

At all points, \( B \) is normal to the surface of the cross-section. The cross-section is defined by the toroid’s inner radius \( r_{in} \), the toroid’s outer radius \( r_{out} \), and its depth (the dimension in the direction of the toroid’s axis), \( h \).

The magnetic field is uniform with respect to the direction of the toroid’s axis, simplifying the area element of the integral to \( dS = h dr \) and the integral to:

\[
\Phi_B = h \int_{r_{in}}^{r_{out}} \frac{\mu_0 I_{loop}}{2\pi r} dr \tag{35}
\]

\[
\Phi_B = h \frac{\mu_0 I_{loop}}{2\pi} \ln \frac{r_{out}}{r_{in}} \tag{36}
\]

So the self-inductance of the toroid is:

\[
L = h \frac{\mu_0 I_{beam}}{2\pi} \ln \frac{r_{out}}{r_{in}} \tag{37}
\]

The mutual inductance, \( M \), between the beam current passing through the toroid, \( I_{beam} \), and the toroid itself describes the ability of the beam’s changing current to induce electromotive forces around the toroid’s cross-section. It is defined as the magnetic flux cutting the toroid’s cross-section per unit beam current passing through the device \[4\]. This may be visualized in Figure 2.

\[
M = \frac{\Phi_B}{I_{beam}} \tag{38}
\]

The magnetic field of this beam current is given by Ampere’s law as being azimuthally and axially symmetric and azimuthally directed around the toroid’s circumference, with magnitude depending only on radial distance from the toroid’s center (where the beam current is located), \( r \):

\[
B_{beam}(r) = \frac{\mu_0 I_{beam}}{2\pi r} \tag{39}
\]

The flux through the toroid’s cross-section from this beam current’s field may be found by integrating:

\[
\Phi_B = \iiint_{\text{cross-section}} B \cdot dS \tag{40}
\]

Substituting Equation 39 and integrating over the area defined by the toroid’s cross-section (between \( r_{in} \) and \( r_{out} \), radially, and \( h \) in the axial direction), we obtain the following expressions for the flux and thus the mutual inductance:

\[
\Phi_B = h \frac{\mu_0 I_{beam}}{2\pi} \ln \frac{r_{out}}{r_{in}} \tag{41}
\]

\[
M = h \frac{\mu_0 I_{beam}}{2\pi} \ln \frac{r_{out}}{r_{in}} \tag{42}
\]

Thus, we see by comparison between Equations 37 and 42 that the self-inductance of the toroid and the mutual inductance between the toroid and the charge beam are equal, \( L = M \), verifying Equation 6.


