UNDERSTANDING THE FORMATION OF YOUNG STARS IN THE CENTRAL 0.5 PC OF THE GALAXY: METHODS FOR EXTENDING THE IMF TO DIM STELLAR POPULATIONS

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ABSTRACT

We test and analyze an existing statistical typing method, and further apply that method to construct a luminosity functions which covers stars down to a luminosity of 16.0 mag. These methods are implemented on data sets collected using the near-infrared integral field spectrograph OSIRIS on Keck II. Previously, the data has allowed for separation of early-type stars (~4-6 Myr) from late-type stars (>1 Gyr) down to $K' = 15.5$ mag. We further confirm the ability of the spectral typing method to accurately differentiate between early- and late-type stars to $K' = 15.5$ mag, and also demonstrate the effectiveness of the method at magnitudes to $K' = 16.0$ mag. In confirming that the statistical spectral typing methods can be extended to dimmer populations, near 16.0 mag, we are able to further establish that the luminosity function of the early-type stars observed at fainter magnitudes indicates a mass function which is less top-heavy than previously derived, but which is still not in agreement with the Salpeter model. These conclusions call for further observational evidence for lower mass stars, beyond 16.0 mag.

Subject headings: Galaxy: center – stars: early-type, late-type, luminosity function, mass function – methods: statistical, spectral typing

1. INTRODUCTION

The properties of the young nuclear star cluster, which surrounds the supermassive black hole, which we call Sagittarius A* (Sgr A*) at the Galactic Center, have been, and continue to be, studied extensively, leading to the discussion of the properties of the nuclear cluster. (e.g. Do et al. 2013; Ghez et al. 2008; Lu et al. 2013) This nuclear star cluster is made up mainly by an old stellar cluster, which extends to a radius of 0.5 pc, and a population of young stars, between about 4 to 6 Myr old, within the central 0.5 pc. These young stars dominate the luminosity of the inner region of the Milky Way. (Do et al. 2013)

The presence of this young star population in such close proximity to the black hole allows for the study of star formation in what is probably the most extreme environment in our galaxy. According to the formation processes with which we are currently familiar, the gases which form these early-type stars should have been shredded apart before any extensive accretion could take place. Yet, it is clear, due to a number of stars with bright emission lines, that a distribution of young stars lies within the central 0.5 pc (Krabbe et al. 1991). However, because this region contains so many dim stars, it is difficult, and takes a considerable amount of time, to distinguish and resolve the individual spectra of the faint, early-type stars. With the application of adaptive optics technology and integral-field spectroscopy, it is possible to separate the young and the old populations. (Do et al. 2013)

The old stars in the Galactic Center, tend to be bright, and massive enough to be manually spectral typed. This is not true for the young nuclear cluster, especially for the dimmer population. Manually typing the spectra of those stars is impossible, so we have created a statistical method which is capable of typing the dimmer spectra accurately. It is important to understand to what extent we trust that statistical method used in determining which stars fall among the early-type population, and which fall within the late-type population. We begin with further applications of the Bayesian inference method presented in Do et al. (2013).

In continuing with our utilization of applications of the Bayesian inference method discussed previously, we determine the algorithm’s ability to assign a probability to a weighted, stacked spectra of stars with magnitudes between $K' = 15.5$ and 16.0 mag, for both early- and late-type stars. A successful result gives motivation to extend other luminosity and mass functions to dimmer stellar populations.

In studying the stars which lie near the central supermassive black hole, we are also especially interested in the luminosity function, and thus the initial mass function (IMF), of the nuclear cluster. The luminosity function itself gives the distribution of luminosities within the region. However, it can be used to determine other properties of the cluster, such as age and IMF. (Do et al. 2013) Studying the IMF is of incredible importance, as it might lead to the affirmation of certain proposed star formation theories, like those presented in Genzel et al.
This close study of the IMF in the Galactic Center will help astronomers to define the formation history of the region, a process not presently understood.

It is important, in order to more accurately constrain the IMF of the Galactic Center, that we be able to extend the predicted luminosity functions down to dimmer stellar populations. By only including stars with magnitudes to 15.5 mag, we only extend to stars with a minimum mass of about \(13 M_\odot\). If we are able to extend the IMF to 16.0 mag, our survey would include stars with masses as low as \(7 M_\odot\). (Lu et al. 2013) By observing fainter magnitude stars, and thus less massive stars, we are adding more stars to our survey, as stars in each luminosity bin increases with decreasing luminosity. Because this faint end of the KLF is where we see the most noticeable difference across the models, we will be able to constrain the model that follows the Galactic Center IMF. (Do et al. 2013)

In this work, we present our application of the statistical spectral typing method to existing OSIRIS field data, for stars up to \(K' = 16.0\) mag. In Section 2, we briefly discuss the parameters of the data collection process for this data utilized here. In Section 3, we discuss our evaluation of the statistical typing method presented in Do et al. (2013). In Section 4, we discuss the implications for extending the IMF to dimmer populations of stars, and finally, in Section 5 we review our conclusions.

2. DATA

All data utilized in this work were obtained with the near-infrared integral field spectrograph OSIRIS on the Keck II telescope, and includes observations reported in Do et al. (2009a), Do (2010), and Do et al. (2013). The overall spectroscopic survey of the region is known as the Galactic Center OSIRIS Wide-field Survey (GCOWS). Observations of these spectra are taken through a Kn3 filter, covering the wavelengths from 2.121 to 2.230 \(\mu\)m (W. M. Keck Observatory 2011). The details of this survey are given in Do et al. (2013).

3. TESTING THE BAYESIAN METHOD

The method we are applying to this data was derived for the purpose of differentiating main sequence, early-type stars, from those of later spectral types. This section explains the process of the Bayesian inference algorithm.

3.1. Classifying Stars

Stars are classified under the hypotheses that they are early-type (\(H_E\)) or late-type (\(H_L\)). The algorithm systematically assigns each star a probability of being either early-, \(P_E\), or late-type, \(P_L\). First, each star is assigned as either early-type, where \(P_E = 1\), late-type, \(P_L = 1\), or untyped, according to the following criteria: 1) Any star with significant Na I features (2.05 and 2.210 \(\mu\)m) is classified as late-type; 2) any star with a Br\(\gamma\) feature (2.165 \(\mu\)m) and no Na I features are classified as early-type; 3) stars with \(K' \lesssim 13.0\) mag and featureless spectra between 2.121 and 2.220 \(\mu\)m are classified as early-type; 4) any remaining star with unclear spectral features is classified as untyped; and 5) stars with signal-to-noise ratios (SNR) < 5 are also classified as untyped. Next, using the manually typed stars, the Bayesian algorithm is trained to recognize the properties of young and old spectra. Finally, untyped sources are assigned probabilities based on the Bayesian evidence, using the training sample, as well as star planing simulations (Do et al. 2013). In training the algorithm, all manually typed stars with \(K' > 14.0\) mag are used to construct distribution functions for the equivalent widths of the Br\(\gamma\) and Na I features in early- and late-type stars, respectively. After constructing each equivalent width distribution, a Gaussian is fit to the peak of both distributions, and is used further in the Bayesian analysis. Essentially, the Bayesian algorithm compares the hypotheses: \(H_E\) the star is early-type to \(H_L\) the star is late-type. The relative strengths of the hypotheses are compared and a probability is assigned to a given star. The full statistical process for deriving the Bayesian evidence is outlined in Do et al. (2013). Figure 1 shows the spatial distribution of only the manually typed stars around the Galactic Center.

3.2. Testing the Algorithm

In order to ensure that this Bayesian algorithm has been appropriately calibrated, we further apply the resulting probabilities to our construction of stacked spectra of the early- and late-type stars found at the Galactic Center.

For this process, young stars are selected only under the following constraints: 1) the assigned probability of the young star \(P_E\) must fall within a certain range; and 2) the SNR must be greater than 5. For example, stars might have been selected under the condition that \(0.75 < P_E < 1.00\). For the purpose of testing the algorithm’s calibration over a range of probabilities, the conditions placed on \(P_E\) are selected individually for each assessment of the algorithm. We choose to select only those stars with sufficiently high SNR. By including those stellar spectra with lower SNR in our selected stars, the stacked spectra would have too much noise to produce any conclusive evidence of the success or failure of the Bayesian method.

Before compiling any weighted spectra, it is important to remove any noise across the spectra which might be caused by single bad pixels or other rare outliers in the spectra. \(\sigma\)-clipping is useful for eliminating such data. The \(\sigma\)-clipping process was carried out in this matter:

1. At each wavelength, across the spectra of all of the selected stars, the mean, \(\bar{f}\), the median, \(\tilde{f}\), and the standard deviation of the flux, \(\sigma_f\), are calculated.

2. Flux values which are less than \(\bar{f} - 3\sigma\) are removed and replaced with \(\tilde{f}\).

3. Flux values which are greater than \(\bar{f} + 3\sigma\) are also replaced with \(\tilde{f}\).

For our purposes, the median, \(\tilde{f}\), is used as our measure of central tendency, rather than the mean. Since it is likely that at least one of the selected stars has some flux value which lies significantly far from the mean (at least \(4\sigma\)), the mean will be significantly affected by the presence of this outlier, whereas the median will be
affected to a lesser degree. A single iteration of this clipping is carried out for the selected set of stars. Each σ-clipped spectra is then weighted by its probability of being a young star ($P_E$). The individual, weighted spectra are summed, and divided by the sum of all of the star’s probabilities of being young.

$$\frac{\sum_i P_{E,i} \text{(Individual young spectra)}}{\sum_i P_{E,i}}$$  \hspace{1cm} (1)

The resulting spectra (Eqn. 1) is a weighted average of all spectra of stars with $P_E$ between $P_{E,\text{min}}$ and $P_{E,\text{max}}$. The same process is carried out for the late-type stars, where the assigned probability of the old star ($P_L$) falls within the range of $P_{L,\text{min}} < P_L < P_{L,\text{max}}$, and the resulting spectra are calculated according to Eqn. 2.

$$\frac{\sum_i P_{L,i} \text{(Individual old spectra)}}{\sum_i P_{L,i}}$$  \hspace{1cm} (2)

### 3.3. Probabilistic Spectra

The resulting weighted average spectra for both the early- and late-type stars have been constructed for the probability ranges $0.00 < P_E \leq 1.00$ and $0.00 < P_L \leq 1.00$, and are displayed in Figures 2 and 3, respectively. Other spectra for some intermediate probability ranges are displayed in Appendix A. The Br $\gamma$ and Na I wavelengths are marked on each plot, as well as the CO lines.

Without performing any unnecessary calculations, we notice the expected Br $\gamma$ feature in the spectra of Fig. 2, as well as in those spectra weighted with $P_E$ presented in Appendix A (Fig. 9, 10, and 11). These resulting spectra have no clear sodium lines or other spectral features.

In Fig. 3, and in the spectra weighted with $P_L$ in Appendix A (Fig. 12, 13, and 14), the resulting spectra have clear sodium doublets between the wavelengths of 2.205 and 2.210 $\mu$m.

Since each spectra is weighted by its assigned probability before being added to the early- or late-type stacked spectra, those stars with low $P_E$ (or $P_L$, in the case of the old stacked spectrum) have a lesser effect on the width and depth of the Br $\gamma$ (or Na I) feature, and on the general shape of the spectrum. As we extend our probability ranges down to $P_{E,\text{min}}$ less than 0.50, we see minute changes in the stacked spectrum. At this point, because the stacked spectrum already contains the spectra of stars with significantly higher values for $P_E$ or $P_L$, those spectra with lower assigned probabilities will contribute very little to the general shape of the stacked spectrum. This explains why we see essentially no shape change between the spectra in Fig. 2 and 11, or between the spectra in Fig. 3 and 14. We see, perhaps, a reduction in the noise of each of these spectra as we increase the probability range over which we select our set of stars, but the underlying spectra remains relatively unchanged.

By comparison of the spectral features in the spectra produced by the method explained in Section 3.3 to those features of the manually typed sources found within the Galactic Center fields, it is clear that the Bayesian method presented in Do et al. (2013) is an accurate and appropriate method for determining the approximate age of the early- and late-type stars detected within the inner 0.5 pc of the Milky Way. This algorithm should be used in future spectroscopic surveys, and may be applied to dimmer stellar populations, in order to fully resolve the spatial distribution of early- and late-type stars at the Galactic Center.

### 3.4. Application to Dimmer Populations
Fig. 2.— Weighted average spectra of all stars with $0.00 < P_E \leq 1.00$. This stacked spectra contains the spectra of 154 stars. The expected, early-type feature (Br $\gamma$) is marked by a solid black line at 2165 nm.

Because the extension of the KLF to dimmer magnitudes is important to resolve the IMF at the Galactic Center, we must first ensure that the algorithm itself is capable of differentiating early- and late-type stars. This distinction is fundamental to our ability to produce an exact luminosity function.

For this process, stars are selected under the condition that they fall within the magnitude range $15.5 \leq K' \leq 16.0$ mag. We include all stars which fit this condition, and do not discriminate based on SNR. Because there are so few stars available to us in this magnitude ranges, selecting by high SNR would eliminate most of our usable data. We repeat the remaining part of the process outlined in Section 3.2, which includes $\sigma$-clipping the data to remove bad pixels, etc., and weighting and stacking the spectra. Two separate spectra are compiled: the first for the early-type stars, where each spectra is weighted by $P_E$; a second for the late-type stars, with weighting by $P_L$.

The resulting weighted average spectra for both the early- and late-type stars have been constructed for the $15.5 \leq K' \leq 16.0$ mag magnitude bin. The stacked spectra are displayed in Figures 4 and 5, respectively. The Br $\gamma$ and Na I wavelengths are marked on each plot, as well as the CO lines.

We again notice the expected Br $\gamma$ feature in the spec-
Fig. 4.— Weighted average spectra of all young stars with $15.5 \leq K' \leq 16.0$ mag. This stacked spectra contains the spectra of 43 stars. The manually typed star, S0-14, is plotted over the weighted spectra for reference to a young stellar spectra. For a plot of the stacked spectra without this reference star, see Appendix A.

Though the spectrum is noisy, we are able to run our spectral typing algorithm on this resulting stacked spectrum. The calculation is carried out in a slightly different manner, as the algorithm typically requires some other parameters about an individual star, such as its location in relation to Sgr A*, but since we have taken a weighted average of a set of spectra, we have lost this information: however, the algorithm can still compute the probability of the resulting spectrum based purely on the features of the spectra. The algorithm returns a probability $P_E = 0.9802$ for the spectrum in Fig. 4. For the spectrum in Fig. 5, we observe a prominent sodium feature, and the algorithm computes a probability of $P_L = 0.9852$.

As noted previously, the algorithm presented by Do et al. (2013) was able to accurately differentiate early- and late-type spectra, and assign appropriate probabilities to those spectra based on their observed features. We now note that the algorithm's ability to differentiate between young and old stars does also extend to very dim populations. The Bayesian algorithm has been thor-
oughly trained by star planting simulations, and as such, is capable of discerning the young from the old, even at magnitudes to 16.0 mag. If the method is to be applied to even dimmer, less massive populations, further analysis is required.

4. GALACTIC CENTER POPULATION MODELS

It is clear that the statistical typing algorithm presented in Do et al. (2013) is capable of determining whether a star is young or old, thus it is appropriate for us to extend the proposed IMF models to the next lowest magnitude, and compare those models with what has been observed.

4.1. Observed Distribution

In creating an observed distribution for comparison to the Salpeter, Top-Heavy (Bartko et al. 2010), and Lu et al. (2013) models, we will extend our observations to the 16 mag bin, whereas most other models have only extended their predictions to 15.5 mag. The distribution of stars derived from our database is shown in Fig. 6. Each star which has not been manually typed within a certain magnitude bin, is given a weight equal to its $P_E$. Any star which is a manually typed star is given a weight of 1. The distribution is normalized over the bins shown. Note that this distribution has been extended to 16.0 mag.

![Fig. 6.— Distribution of observed early-type stars in the Galactic Center nuclear cluster. Data previously presented in Do et al. (2013).](image)

Though we believe that we can trust the algorithm’s ability to classify stars and early- or late-type, we need to also be cautious of our own observational abilities. Table 1 gives the completeness for each field of the Galactic Center fields. By correcting for low completeness values, we eliminate those fields in which only some sources can be observed. The completeness values in the table given the ratio of manually typed stars to all detected sources in that magnitude bin, in that field. For example, in the 15.0 to 15.5 mag bin, the NE field has a completeness value of 0.50, meaning that only half of the stars that have been observed which fit both of those parameters have been manually typed. Though this is a relatively low completeness value, when compared to the bins which have 1.00 completeness, for our sample, it is sufficient.

To account for our own observational incompleteness, we will create a second distribution of stars, by selecting only from fields in which we have been able to manually type a significant number of stars. For our own purposes, any field which has a completeness below 0.30 at magnitudes greater than 15.0 will be excluded from our second sample of stars. For a completeness minimum of 0.30 at magnitudes above 15.0, nine of the available fields will remain in our sample. Following the same process as before, each star in the remaining fields is weighted either by $P_E$, or by 1 if the star has been manually typed. The distribution is again normalized across the bins, and is shown, along with the distribution which does not correct for completeness, in Fig. 7.

![Fig. 7.— Distribution of observed early-type stars in the Galactic Center nuclear cluster for fields with completeness greater than 0.30. The remaining fields are the Central, SE, S, N, E2-3, E3-1, E3-2, E3-3, E4-2, and E4-3 fields.](image)

A $\chi^2$-test between these two distributions returns $\chi^2_{\text{red}}$ value of 0.713, where an ideal $\chi^2_{\text{red}}$ value is 1. For ten degrees of freedom ($N - n - 1$, where $N$ is the number of observations, 11 bins, and $n$ is the number of free parameters, which is zero in this case), this $\chi^2_{\text{red}}$ value corresponds to a p-value of 0.713. Even at a significance level of 0.10, we are very certain that both of these distributions come from the same original sample population. We can continue our comparison of models to our observed distribution using the original observed distribution, which does not select out fields based on
completeness values (i.e., the distribution first displayed in Fig. 6).

4.2. Comparison to Models

Our final step is to compare the observed KLF (see Fig. 6) to those proposed models for the KLF (see Fig. 8). The Salpeter KLF corresponds to an IMF with slope $\alpha = 2.35$, which was computed by Edwin Salpeter in 1955. The Top-Heavy IMF was proposed in Bartko et al. (2010) and corresponds to an IMF slope of $\alpha = 0.45$. The Lu model is derived from a simulated nuclear cluster, with an IMF slope of $\alpha = 1.7$. We are most concerned with comparing the simulated cluster, presented in Lu et al. (2013), to our observed distribution, and checking for agreement between the two data sets. We are also especially concerned with the variance between models in the very dim magnitude bins. If, at magnitudes above 15.0 mag, we find agreement between our observed distribution and one of the models, we can, with higher certainty, distinguish that model as a more probable model for the actual IMF at the Galactic Center, as at fainter magnitudes, we notice a significant divergence in the models.

Because the Lu et al. (2013) simulated cluster is derived from the same data which we present in the observed distribution, we expect the highest p-value to be returned from the $\chi^2_{red}$ test between these two distributions. If we consider the simulated cluster to be the expected distribution, we determine a $\chi^2_{red}$ value of 3.132. With ten degrees of freedom, as in the previous $\chi^2_{red}$ test, we derive a p-value of 0.978.

We can also take the Salpeter model to be the expected distribution, and again conduct a $\chi^2_{red}$ test to determine if this model is representative of the observed distribution. The test returns a $\chi^2_{red}$ value of 7.325, which corresponds to a p-value of 0.694.

In our last test, we take the Top-Heavy model, as presented in Bartko et al. (2010), as the expected distribution. The test gives a $\chi^2_{red}$ value of 7.105, and a p-value of 0.715.

4.3. Discussion

As expected, the observed distribution, as suggested by the p-value, is consistent with the data presented by the simulated cluster. However, it is also interesting that the p-values calculated by comparison of the observed distribution to the Salpeter and Top-Heavy models suggests a lesser degree of agreement between the observed distribution and the models. The p-value does not fall below a significance level of 0.10, so we cannot absolutely reject these models as potential IMF models for the Galactic Center. It is important to note though, that the model suggested by an IMF slope of $\alpha = 1.7$, as presented in Lu et al. (2013), is most representative of the observed cluster. It is equally as important to note that in the magnitude bin covering 15.5 to 16.0 mag the Salpeter and Top-Heavy models deviate noticeably from the observed distribution.

It is clear from this study that further observations of the Galactic Center are necessary to truly understand which of these IMF models are representative of the actual IMF. If, in further study of dimmer stellar populations, we find that the observed cluster continues to depart significantly from either the Salpeter or Top-Heavy IMF models, especially in the low magnitude bins, we will be able to reject one, or both, of these models in favor of a model with an intermediate IMF slope, such as that suggested by the findings of Lu et al. (2013).

5. Conclusions

We test and analyze the existing statistical typing method, as outlined in Do et al. (2013), and apply that method to construct a luminosity functions which extends to stars with masses down to $7 M_\odot$. This mass corresponds to a magnitude of 16.0 mag. We are able to confirm the ability of the spectral typing method to accurately differentiate between early- and late-types stars, to $K^\prime = 16.0$ mag. In again testing the spectral typing algorithm, we find that the algorithm assigns $P_E = 0.9802$ to the weighted spectra of all of the early-type stars at the Galactic Center which fall in the 15.5 to 16.0 mag bin, and $P_L = 0.9852$ to the weighted spectra of the late-type stars in the same bin.

In confirming that the statistical spectral typing method can be extended to 16.0 mag, we are able to further establish that the KLF of the early-type stars observed at fainter magnitudes indicates an IMF which is less top-heavy than previously derived by Salpeter, but is still steeper than the IMF derived by Bartko et al. (2010).

Further measurements of the luminosity function at the Galactic Center is important to our understanding of the IMF and of the formation processes which might occur in the region surrounding Sgr A*. Work should be continued in extending observations to $K^\prime > 16.0$ mag, as these observations will allow in depth studies of the star forming region down to the solar mass scale, and may lead to some clearer evidence which favors a certain IMF slope.
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APPENDIX

FIGURES & TABLES

Fig. 9.— Weighted average spectra of all stars with $0.75 < P_E \leq 1.00$. This stacked spectra contains the spectra of 18 stars.

Fig. 10.— Weighted average spectra of all stars with $0.50 < P_E \leq 1.00$. This stacked spectra contains the spectra of 35 stars.
Fig. 11.— Weighted average spectra of all stars with $0.25 < P_L \leq 1.00$. This stacked spectra contains the spectra of 69 stars.

Fig. 12.— Weighted average spectra of all stars with $0.75 < P_L \leq 1.00$. This stacked spectra contains the spectra of 169 stars.

Fig. 13.— Weighted average spectra of all stars with $0.50 < P_L \leq 1.00$. This stacked spectra contains the spectra of 202 stars.
Fig. 14.— Weighted average spectra of all stars with $0.25 < P_L \leq 1.00$. This stacked spectra contains the spectra of 222 stars.

Fig. 15.— Weighted average spectra of all young stars with $15.0 \leq K' \leq 15.5$ mag. This stacked spectra contains the spectra of 36 stars. The algorithm returns a probability of $P_E = 0.9911$ for this spectrum.
Fig. 16.— Weighted average spectra of all old stars with $15.0 \leq K' \leq 15.5$ mag. This stacked spectra contains the spectra of 131 stars. The algorithm gives a probability $P_L = 0.9997$ for this old spectrum.

Fig. 17.— Weighted average spectra of all young stars with $15.5 \leq K' \leq 16.0$ mag. This stacked spectra contains the spectra of 43 stars. The probability for this spectra to be young is given as $P_E = 0.9802$ from the algorithm.
Fig. 18.—Weighted average spectra of all old stars with $15.5 \leq K' \leq 16.0$ mag. This stacked spectra contains the spectra of 121 stars. The probability given by the algorithm for this old star is $P_L = 0.9852$.

TABLE 1

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Note. — In Table 1, the magnitude bins are representative of the observed $K'$ magnitudes. The table contains the fraction of stars that have been manually spectral-typed out of all sources detected in that magnitude bin. (Do et al. 2013)