Eccentric Behavior of Eccentric Planets

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Abstract

Recent increases in the discovery of high multiplicity Sub-Jovian planetary systems, in addition to discoveries of giant planets of high eccentricities in these multiple body systems, lead to questions into their origins and evolution. The study of secular interactions is not new, however it has only recently been applied to exoplanetary systems. These systems sometimes fall far from the classical, solar system approach, of low eccentricities and low inclinations. Here I investigate a system of two Jupiter-mass planets and a test particle around a Sun-like star to examine if the classical secular perturbation model still applies when examining a system with high perturber eccentricities. Additionally, I compare this model to the Kepler-11 system of six bodies to simulate the evolution of this system after introducing two Jupiter-sized planets into the Kepler-11 system.

1. Introduction

Over the past five years, discoveries of exoplanets has vastly increased in volume from year to year. Presumably the detection of exoplanets will continue to increase as detection methods and equipment increase in accuracy. Many of the newly discovered exoplanets exist in orbits and systems unlike our own. The orbits in our solar system have low eccentricity and low inclination, excepting the case of Mercury, and the gas giants are located outside of five astronomical units. The current surveyed extrasolar systems tend to be giant planets much closer to the star, often closer than the Earth, called hot Jupiters. These hot Jupiters also tend to have high eccentricities, which has been a largely unexplored occurrence. In addition, some of the discovered extrasolar systems have a high planet multiplicity; systems with five of more planets are quite common and their secular interactions become important in determining their behavior. This expanding collection of exoplanets prompt questions on the formation and evolution of such eccentric, high multiplicity systems.

Here I examine the extrasolar planetary systems interactions and their evolution, purely with respect to their eccentricities, within the context of the secular approximation. The secular approximation allows for investigations into the exchange of eccentricity and inclination between the bodies in planetary systems over long, secular timescales. The approximation relies upon the assumption that the variations in the gravitational potential that drive the interactions average out over short, orbital timescales. This assumption allows for the system to not exchange energy, and consequently shift their semi-major axes, but instead exchange angular momentum, which in turn exchanges inclination and eccentricity between bodies. The investigation of secular interactions between planets has been studied for hundreds of years, but mostly in the context of our nice solar system. The two methods this paper uses to model planetary interactions are the classical method that dates back to Poincare, and Gauss’s averaging method.

2. Secular Approximation Methods

2.1. Classical Method

The Classical Method, laid out in Murray and Dermott’s Solar System Dynamics, uses the disturbing function to describe the perturbations on the orbits of a three body system. The disturbing function, in effect, is a function to describe the accelerations of secondary masses in a system relative to the primary by taking
the gradient of the perturbing potential. Murray and Dermott provide a lengthy expansion of this function, however to give a suitable approximation for finding an analytical solution to a particular form of the N-body problem that is secular perturbation theory, only the purely secular terms of the disturbing function are used. They then use the secular terms of the disturbing function to produce matrices describing the two planets’s frequencies as functions of their masses and fixed semi-major axes.

Next, the authors use equations of motion and each planet’s eccentricities to get the eigenfrequencies of the matrices to find the classical Laplace-Lagrange secular solution to the secular problem. The solution described by this method functions under the assumptions that the are no mean motion commensurabilities, the to orbiting bodies’s orbits do not cross, and that the eccentricities and inclinations are small enough that the used expansion of the disturbing function is sufficient to describe the motion.

2.2. Gauss’s Averaging Method

Gauss’s averaging method, instead of using the disturbing function expansion to find the secular perturbations, considers the effect of perturbations from the external bodies can be found by spreading the external body’s mass around its orbit in a ring of material. The ring’s density is found by smearing out the mass so that the ring mass in a fixed time interval is the same at all points in the orbit. Thus, the ring is densest at apocentre and least dense at pericentre. The dependence is given here:

\[ \rho = \frac{mr}{2\pi a^2 \sqrt{1-e^2}} \]  

The mass of the ring exerts a force on the inner body, which can be used to derive the precession rate of the inner body. This is explicitly shown in Murray and Dermott and is proved to be the same result as the classical method for small eccentricities. The implementation of this method was created previously in Will Farr’s code, which applies the aforementioned method to an N-body system.

3. Methods

Using the classical case of the Sun, Jupiter, Saturn, and a test particle at an arbitrary semi-major axis, internal to the other two, I found the locations where the eccentricity of the particle became excited over secular timescales. I attempted to reproduce the results of the locations of the eccentricity peaks by using Gauss’s averaging method to compare the two methods for low eccentricity perturbers in a stable system. Because the two methods share the same results for small eccentricities, there was little variance between the two.

After finding the planets’ secular effect on each other, I decided to mimic Murray and Dermott’s approach by placing a test particle within the orbit of Jupiter and Saturn with both methods to see if the predicted peaks at the eigenfrequencies of the classic method matched with those of the full, Gauss’s averaging method. Again, since Jupiter and Saturn have low eccentricity, the results matched up well. The only major divergence between the two comes from computing the secular evolutions for a test particle at a semi-major axis near the eigenfrequencies. The difference comes from the amount of computing time needed at each point to evolve the system for long enough to finally reach a resonance that would make the particle’s eccentricity large enough to match the classically predicted value.

The next step I took was to look at a system of perturbers, a Sun-like star, and a test particle with the perturbers closer to the star, similar to those found in extrasolar systems. Initially, I arbitrarily placed two Jupiter-massed planets at 1 AU (astronomical units) and 3 AU, and assigned them low eccentricity so as to match the classical situation. I found a similar match to the classical method when using Gauss’s averaging method, the same computational divergence occurred, again because it would take too much computing time to analyze to times where such high, predicted eccen-
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Eccentricities would occur. After testing the classical situation, I began systematically increasing the eccentricity of the perturbers, simultaneously, to $e=0.1$, then increasing both by 0.1 until reaching an eccentricity of $e=0.5$ for both perturbers (See Figure 1). From the figures it is evident that as the eccentricity of the perturbers increased, the test particle’s maximum eccentricity varied more erratically than predicted by the approximation. The reason I stopped at an eccentricity of 0.5 was because the test particle was very rapidly breaking the approximation that the orbits would not cross, in addition to a large increasing in computing time for such high eccentricities.

I repeated the same process of comparing the two methods for a system of two perturbers bodies of varying eccentricity, this time for perturbers placed at 3 AU and 5 AU. The results were similar with respect to higher eccentricities causing deviations from the classical model. The new perturber distances however, did seems to excite a wider range of test particles’ semi-major axes and began to deviate from the classical model at lower eccentricities (See Figure 2). To examine the effects of two perturbers in an actual extrasolar system, I looked at an already stable, well-studied system, Kepler 11. Kepler 11 contains a Sun-like star with six planetary bodies ranging from about Earth to Neptune masses orbiting between 0.09 AU and 0.466 AU. The eccentricities of these bodies do not exceed 0.05, which is about the eccentricity of most of the planets in the Solar System. Presumably placing perturbers in this system would be the same as the previous experiment of placing test particles at the specific semi-major axes of the planets.
There is an additional effect of the individual planets affecting each others’ eccentricity through exchanges of angular momentum instead of just having a system of four particles (the star, the test particle, and two perturbers), now Gauss’s method is applied to a system of nine bodies, all interacting with each other.

As with the previous experiments, I began by placing two Jupiter-sized planets at 1 AU and 3 AU with an initially low eccentricity ($e=0.05$). The system remained stable and no crossing of orbits occurred since the planets’ time evolution remained fairly unexcited (Figures 3 and 4). As before, I increased the eccentricity of the two perturbers, simultaneously, until the two innermost planets’ orbits began to cross, thus breaking our secular approximation that the energies remain the same over orbital timescales (Figures 5 and 6).

I decided to move the perturbers to 0.7 AU and 3.7 AU, the distance being arbitrary, except to try and prevent mean motion resonances, which our code does not take into account. I started with an eccentricity of 0.05 again to see how this new system would respond to the different perturbing locations (Figures 7 and 8). The individual oscillations inside the larger envelope seemed to increase, but the amplitude of the maximum eccentricities remained low. When the eccentricities of the perturbers were boosted, the inner two planets of Kepler 11 began to cross orbits earlier ($e=.15$) than in the previous case of perturbers at 1 AU and 3 AU (Figures 9 and 10).

4. Conclusions

From the comparisons between Gauss’s averaging method and the classical method it preliminarily appears that for perturbers with eccentricities greater than around 0.1, the peaks of the secular resonances fall at different semi-
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Figure 3: Time evolution of the Kepler 11 system with two, low-eccentricity perturbers at 1 AU and 3 AU major axes than the classical model predicts. This conclusion has been drawn from two sample cases and no statistics on the actual deviation from the predicted peak, therefore more testing is needed. The reason there is some weight to this claim however, is that we only looked at the case where inclination was zero in this paper. After a few trials with both eccentricity and inclination, not shown here, the plots for maximum eccentricity began to look much different than the classical model. The second peak in particular would decay slowly in several smaller peaks when inclination was included into the perturbers and test particle system. Further work could be done to look into similar tests to those done here, but including inclination to see how it would affect the same type of system.

When looking at the two simulations of the Kepler 11 system with two perturbers, it appears that for the low-eccentricity case, it is possible to hide two larger planets outside of the range where most modern exoplanetary detecting methods are capable of looking. This could be helpful for explaining planetary formation by using this and further results to support theories that it is possible to form large planets and then form several inner, terrestrial planets without ejection. These results do indicate that planets such as those in the Kepler 11 system that are in close proximity to each other cannot withstand eccentric outer perturbers.

References
Figure 4: Orbit crossing of the Kepler 11 system with two, low-eccentricity perturbers at 1 AU and 3 AU
Figure 5: Time evolution of the Kepler 11 system with two, high-eccentricity perturbers at 1 AU and 3 AU

Figure 6: Orbit crossing of the Kepler 11 system with two, high-eccentricity perturbers at 1 AU and 3 AU
Figure 7: Time evolution of the Kepler 11 system with two, low-eccentricity perturbers at 0.7 AU and 3.7 AU

Figure 8: Orbit crossing of the Kepler 11 system with two, low-eccentricity perturbers at 0.7 AU and 3.7 AU
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Figure 9: Time evolution of the Kepler 11 system with two, higher-eccentricity perturbers at 0.7 AU and 3.7 AU

Figure 10: Orbit crossing of the Kepler 11 system with two, higher-eccentricity perturbers at 0.7 AU and 3.7 AU