ABSTRACT

We present the results from testing the possibility of measuring differential parallax signals from stars in the direction of the Galactic Center and extracting accurate distances. This results were achieved by modeling the motions of stars due to their differential parallactic and proper motion and then simulating data. We varied the distance to the star and the measurement error in each simulation in order to obtain a limitation for detecting parallactic motion and obtaining accurate distances measurements with current and future observations. With the current measurement error of 0.1 mas, the differential parallactic signal can be detected at a distance of 4 - 5 kpc.

1. Introduction

The Galactic Center plays host to the nearest supermassive black hole (SMBH) to our Galaxy. Evidence of the existence of the SMBH was produced by tracking the kinematics of stellar point sources in the Galactic Center (Eckart & Genzel 1997; Ghez et al. 1998). Figure 1 labels the location of the supermassive hole. Since its discovery, there have been numerous studies to make precision measurements of the
mass, radius and distance of the black hole. The current estimates of the black hole’s mass and distance are $4.1 \pm 0.6 \times 10^6 M_\odot$ and $8.0 \pm 0.6 \text{kpc}$ (Ghez et al. 2005).

Fig. 1.— An HKL-band color mosaic of the Galactic Center. The white arrow marks the position of the supermassive black hole. Credit: UCLA Galactic Center Group

Adaptive Optics (AO) systems have been crucial in obtaining high-resolution data of the Galactic Center. AO systems allow the telescopes to reach the diffraction limit as they correct for atmospheric turbulence. This is particularly key for larger telescopes, such as the Keck telescopes, as their diffraction limit will be smaller than small telescopes ($\theta \sim \lambda/D$). As shown in figure 2, in the absence of atmospheric turbulence the wavefront of a star can be focused, so the light from star becomes more point-like. In reality, atmospheric turbulence will distort the wavefront and as a result the wavefront will not focus, causing the image of the star to appear blurry (Hardy 1994).
Fig. 2.— Left: The perfect point spread function of a star without atmospheric turbulence. Right: The distorted point spread function of a star who's wavefront has been affected by atmospheric turbulence. Credit: Hardy (1994).

AO corrects for wavefront distortions by using a wavefront sensor to measure the incoming wavefront and then moves a deformable mirror to correct for any wavefront error. Since the atmosphere is constantly changing, the wavefront distortion is measured and corrected at about 100 times a second (Wizinowich 2005). One challenge to this technique is that it will only work if there is a bright star in the same small field of view that is being observed. If the bright star is far from the source of interest then the wavefronts will be affected differently since the atmospheric turbulence in two parts of the sky are not necessarily the same. To account for this problem, the Keck telescope AO system uses a laser guide star (LGS) system. The Keck II LGSAO system has greatly improved the quality of images of the Galactic
Center (Ghez et al. 2008). Figure 3 shows the images of the Galactic Center with and without the AO system.

Even though the Keck LGSAO system has made it possible for us to clearly image the Galactic Center with great resolution (as seen in figure 1), distances to these stars is still very difficult to measure. In general, distance is one of the most difficult parameters of astronomical objects to measure. The first distance measurement made to planets was done using a technique called trigonometric parallax. The distance to a planet was indirectly measured by observing the planet at two distant locations on Earth and measuring the angular position at each location. Then through simple trigonometry the distance to the planet can be calculated.

Using a similar method called stellar parallax, the distances to stars can be determined. The general idea of this method is that two observations of a star of interest are made six months apart therefore using a baseline equal to the diameter of the Earth’s orbit. If the star is nearby then its position will seem to change with respect to the distant stars, which seem to stationary as illustrated in Figure 4. One-half of total change in the stars apparent position is called the parallax angle, \( p \). Measuring \( p \) allows astronomers to measure the distance, \( d \), to the star.

\[
d = \frac{1}{\tan p} \simeq \frac{206.265}{p''} \text{AU} \tag{1}
\]
where \( \tan p \simeq p \) due to small-angle approximation \( p \) and \( p'' \) is in arc-seconds (Carroll et al. 2007). By converting to distances to parsec (1pc = 2.06 \times 10^5 AU then)

\[
d = \frac{1}{p''} \text{pc}. \tag{2}
\]

Differential parallax is the parallactic angle of a star with respect to another object in the sky. The differential parallactic angle of stars in this study is in the reference frame of the Galactic Center:

\[
d(\text{pc}) = \frac{1}{p''_i} - \frac{1}{p''_{gc}} \tag{3}
\]
where the parallactic angle of the Galactic center \( p''_{gc} \) is 1.25 \times 10^{-1} \text{mas} .

This paper presents simulated data of the motion of stars due to differential parallax. Different distances and order of magnitude measurement errors were tested for
Fig. 3.— Observations of the Galactic Center with and without Adaptive Optics
Credit: UCLA Galactic Center Group
in order to determine the limitation of detecting and measuring differential parallax for current and future telescopes.

2. Test Sample: Foreground Stars

We will be testing this method on a test sample of two foreground stars which were discovered by Wright (comprehensive exam paper). She identified these two foreground in her study of extinction at the Galactic Center. Extinction describes the processes of which light from an astronomical object like stars at the Galactic Center is absorbed and scattered when interacting with interstellar medium. As a result the observed light is reddened compared to the emitted light. In order to measure the extinction $A_v$ at the Galactic center, Wright at el. compared the observed colors of stars at the Galactic Center to their intrinsic color:

$$ (M_{\lambda_1} - M_{\lambda_2})_{\text{reddened}} = (M_{\lambda_1} - M_{\lambda_2})_{\text{intrinsic}} + \frac{A_{\lambda_1}}{A_v} A_v - \frac{A_{\lambda_2}}{A_v} A_v $$

where near-infrared (NIR) extinction law $\frac{A_{\lambda_1}}{A_v} = \frac{A_{\lambda_2}}{A_v}$ is constant. She identified the two foreground sources because their colors are much bluer and their $A_v$ values are...
much smaller compared to stars at the galactic center (see figure 5). The $A_v$ values of the foreground stars are about $5.1 \pm 0.6$ mag and $4.1 \pm 0.6$ mag. By assuming the average extinction rate of the Milky Way of $1\, \text{mag kpc}^{-1}$, the distances to these stars were estimated to by $5.1 \pm 0.6$ kpc and $4.1 \pm 0.6$ kpc.

Fig. 5.— $A_v$ values for Scoville et al. (2003) derived from Paschen $- \alpha$ and 6 cm continuum versus $A_v$ for Wright (comprehensive exam paper) derived from intrinsic colors of stellar sources. The green points are foreground sources. Credit: Wright (comprehensive exam paper)
3. Models

There are a few variables that must be considered when determining the limitation of detecting and measuring with accuracy the differential parallax signals of stars in the direction field of the galactic signal. The first variable is the order of magnitude measurement error in the observation. The value of this variable changes from observation to observation and is affected by the atmospheric conditions of the particular night. On average, the Keck II LGSAO has a measurement error of 0.1 mas for a 14 magnitude star. Figure 6 plots the measurement error versus K magnitude for July 12, 2013. On this particular night, the measurement error was better than the average of 0.1 mas for magnitudes between 10 to 14.

Another variable that affects our ability to measure differential parallax is the distance to the star. As the distance to the star increases, its parallax angle will decrease and can be lower than the measurement error of the observations; at this point, the signal will most likely not be detected. Figure 7 shows how the maximum differential parallax angle varies with increasing distance away from the Earth; as distance increases, the differential parallax angle rapidly decreases, dropping below 0.1 mas at around 4 kpc. It should be noted that a differential parallax angle of 0.1 mas is not our lower limitation. It will be possible to detect a signal that is smaller than 0.1 mas as long as there is a long time baseline since the parallactic signal is repetitive. The horizontal line at 1 mas represents the measurement error of observations that were taken without AO.

The last variable that hinders the measurement of differential parallax is limited observational coverage of the Galactic Center. The UCLA Galactic Center Group has been observing the Galactic Center since 2006 with the Keck LGSAO (see table 1 for observation dates). The Galactic Center is observed from May to August of each year due to the alignment of the Earth’s orbit with respect to the location of the Galactic Center. During the northern hemisphere’s summer, the Earth is in between the Sun and the Galactic Center such that in late June the Galactic Center is at the zenith at midnight. During the winter months, the Galactic Center is highest in the sky during the day, making it impossible to observe. This limited coverage will affect our ability to measure the parallactic signal because only part of the signal is detected.
In order to test the possibility of measuring differential parallax, we modeled the motion of stars due to their differential parallax and proper motion. The models were created in the ecliptic coordinate system and assumed that the stars were only moving in two dimensions. The motion in the y-axis is assumed to only be affected by the proper motion,

\[ y = y_0 + v_y t \]  \hspace{1cm} (5)

and the motion in the x-axis is

\[ x = x_0 + v_x t + A \sin(2\pi + p) \]  \hspace{1cm} (6)

Fig. 6.— Positional Uncertainty (or measurement error) versus Magnitude
Fig. 7.— Differential parallax angle (mas) versus Distance (kpc) away from Earth. The line at 0.1 mas represents the average measurement error of the Keck II LGSAO. The line at 1 mas represents the measurement error of observation of the Galactic Center before AO.

where $p$ is the phase of the signal,

$$A = dp_{\text{max}} = \frac{1}{d_*} - \frac{1}{d_{GC}}. \tag{7}$$

The models and simulations reported in this paper were all done in Interactive Data Language (IDL). To take account of the limited observational coverage a phase of 0 rad was assumed for this paper. In other words, it was assumed that the alignment of the Earth’s orbit was in such a way that the Earth was in between the Galactic Center and our Sun in June. By assuming a phase of 0 rad, our data points would outline the full change in the apparent position of the star, or $2 \times$ the differential parallax of the star. Figure 8 shows two models of the differential parallactic signal of the foreground star at $4.1 \pm 0.6$ kpc (top) and $5.1 \pm 0.6$ kpc (bottom). The red points represent the data points available from our observations. In these models, the proper motion of these stars were ignored.
4. Simulations

Simulations were done to test how distance, $d$, and measurement error, $dx$, affect the detection of the differential parallactic signal. Data sets were created using $RANDOMN$, an IDL function that randomly generates numbers from a Gaussian distribution within 1 $\sigma$ of 0. Data sets consisted of one data point per observation. Each data point was generated by:

$$position_{measured} = position_{model} + dx \times RANDOMN(seed,1),$$

(8)

where $position_{model}$ is the expected position at a certain time of the star calculated by equations 5 and 6.

Each data set was fit by $mpfitfun$, a least-squares regression fitting routine available in IDL. The inputs to this function are the model equations 5 and 6, the data sets, the measurement error and the initial guesses for the parameters. The outputs of each fit are the fitted parameters, their uncertainties, and $\chi^2_k$. In the fitting program, a limit of 0 to 8 kpc was placed on the distance since we did not expect to detect stars past the Galactic Center. A limit of 0 to $2\pi$ was also placed on the phase.

Data sets were simulated for seven distances (1, 2, 3, 4, 5, 6 and 7 kpc) and 200 measurement errors (from 0.01 to 1 mas) for a total of 1400 simulations. The main objective of these observations was to test how the distance and measurement error affects the models and our ability to determine the parallactic angle. Figure 9 shows twenty of the simulations for varying distances and measurement errors. Each plot compares the fitted parameters (the red curve) to the model (the dashed blue line). At further distances (6 to 7 kpc) and larger measurement errors (0.4 to 1.0 mas), the distance output from the fitting program does not agree

| Table 1: Observation Dates per Year using Keck II LGSAO |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
| 5/03 | 5/17 | 5/15 | 5/04 | 5/05 | 5/26 | 5/15 |
| 6/21 | 8/11 | 7/24 | 7/24 | 7/06 | 7/17 | 7/24 |
| 7/17 | —    | —    | 9/09 | 8/14 | 8/22 | —    |
with the inputted distance within error; this is a result of the parallactic signal being too small for the fitting program to fit it properly. Some of the fits did not converge like the fit for $d : dx$ pair 5kpc: 0.7 mas, 7 kpc: 0.7 mas, and 7 kpc: 1 mas. If the signal is too small with respect to the measurement error, then even though it is repetitive, the fitting program will calculate the differential parallax angle to be 0 mas and set the distance to $8 \pm 0 kpc$. 
Fig. 8.— Top: Model of the parallactic motion of the foreground star at $4.1 \pm 0.6$ kpc. Bottom: model of the parallactic motion of the foreground star at $5.1 \pm 0.6$ kpc. The red points represent times where we have Keck LGSAO data.
Fig. 9.— X-position (mas) of a star versus Time. The blue dashed line is the model of the star’s motion. The red solid line is the best fit line from the fitting program. The input parameters are $v = (5, 0)\,\text{mas/yr}$, initial position=$(0, 0)\,\text{mas}$ and phase=0. The input for distance is 1, 3, 5, and 7 kpc. The input for measurement error is 0.01, 0.1, 0.4, 0.7, and 1.0 mas. Each plot includes the output distance of the fit and the $\chi^2_k$. 
5. Discussion

At further distances, a number of fits were not converging. To gain an estimate of the how many fits at certain distances would not converge, one-hundred different data sets were generated and fitted for each distance-measurement error pair. Then the number of non-converging fits were totaled for each measurement error. Figure 10 shows a number of fits that did not converge with respect to distance for the current measurement error. At further distances the fits are less likely to converge due to the small parallactic signal.

![DIagram](#)

**Fig. 10.—** The percentage of fits out of 100 fits that did not converge with respect to distance for measurement error of 0.1 mas

Another fitting parameter that was closely studied was the distance error with respect to measurement error and distance. At the further distances and larger measurement errors, the parallactic signal was small enough that the fitting program could not do a proper job. In other words, the output distance did not agree with the inputted distance within uncertainties. As a result, different data sets with the same input parameters would have very different fitted parameters. In order to get a better estimate of distance error, an average of 100 data sets of the same distance-measurement error pair was taken and plotted against measurement error as shown in figure 11. As distance and measurement error increases, the distance error also
increases since the signal is very small at further distances. Also, the distance error begins to scatter much more for larger distance since those fits are not reliable.

![dD vs dx](image)

**Fig. 11.**— Distance error versus measurement error. Each point is an average of 100 data set of the same distance-measurement error pair.

To take a closer look at what the fits are doing at the current $\Delta x$ of 0.1 mas, distance error was compared to distance (figure 12). From about 1 kpc to 5 kpc distance error rapidly increases with distance, but from 5 to 7 kpc, the curve seems to flatten out, indicating that the fitting program is not properly fitting the signal. The Thirty Meter Telescope is predicted to have a $\Delta x$ of 0.01 mas. At 0.01 mas, the distance error increases with distance without flattening, showing the fits at for 0.01 mas are reliable.

To better understand how distance and measurement error affect the fitted (aka the output) distance from the fitting program, the input distance minus the fitted distance was plotted against measurement error (figure 13). As before, each point is an average of 100 data sets. From 1 to 4 kpc, the fitted distances agree with the
Fig. 12.— Distance Error (kpc) versus Inputted Distance (kpc). The red line represents the distance errors for Keck II LGSAO measurement error of 0.1 mas. The blue line represents the distance error predicted for the TMT measurement error of 0.01 mas.

Input distances, but from 5 to 7 kpc, there is a tendency for the fitting program to underestimate the distance of the star. This is most likely due to the limits put on the distances in the fitting program.
6. Summary

We have presented results of simulations of the motion of star due to differential parallax in order to test the possibility of measuring differential parallax signals. As shown above, with the current measurement error of 0.1 mas the differential parallactic signal can be detected at a distance 4 - 5 kpc away from us. The next step in this project is to measure the change in position of the two foreground stars and see if we can extract an distance using parallax that agrees with the current distances estimates of these stars. Before this can be done, we need simulate the motion of stars at different phases to test how the phase parameter will affect the fits.

There are multiple factors that can improve our ability to detect differential parallax: a longer time baseline, more coverage and better measurement error. A longer time baseline will introduce more data points, increasing the length of the signal. Currently our data is taken from May to August, but we may have the ability to observe from March to September. With more coverage during a year, more of the parallactic signal can be detected. Lastly, as shown above, a better measurement error can significantly increase our distance limitation; at a measurement error of 0.01 mas current simulations have hinted the possibility of measuring the differential parallax signal. Further simulations will be done to fully understand the limitation of 0.01 mas. Better measurement error and observations will come with time as AO is improved and larger telescopes like the TMT come online.

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REFERENCES


Carroll, B. W., & Ostlie, D. A. (2007). The Continuous Spectrum of Light . An intro-


Hardy J. W. 1994, Scientific American, 40


Wright S. A. Comprehensive exam paper

Distance 1.00kpc

Distance 2.00kpc

Distance 3.00kpc

Distance 4.00kpc

Distance 5.00kpc

Distance 6.00kpc
Fig. 13.— Distance$_{\text{input}}$— Average of Distance$_{\text{fitted}}$ (kpc) versus measurement error (mas).