Modeling Light Scattering in Bipolar Nebulae

Z. Boekelheide
Department of Physics and Astronomy, University of California, Los Angeles
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We present a numerical model to image light scattering in bipolar nebulae centered on a luminous star. The model accepts an axisymmetric density distribution as input, and yields an image of a nebula based on that assumed density distribution and a chosen systemic inclinaton. We used the model to find simulated nebulae that resemble the real observed nebulae IRAS22036+5306 and Gomez's Hamburger in their gross characteristics. We also used the model to determine the appearance implied by a theoretical density distribution obtained from an investigation of the hydrodynamics of nebular dust and gas when the mass-losing central star is in a binary system.

Planetary or protoplanetary nebulae are created when red giant or AGB stars eject mass in the form of gas and dust. Red giant and AGB stars are swollen to many times their main sequence size, and particles in the outer layers are only loosely gravitationally bound. Radiation pressure from the star acts on these particles and pushes them out, so the central star is surrounded by outward-flowing gas and dust. Light from the star scatters on the dust and the image seen from a telescope is a reflection nebula. In the case of bipolar nebulae, the gas and dust are organized in an axisymmetric distribution. We created a computer program to model the scattering of light through the dust in a bipolar nebula, for a given density distribution and angle of inclination with respect to the viewer. The product of such a simulation is an image showing the appearance of a nebula with the given distributions and angle of inclination.

Although the reasons for nebula formation in general are fairly well known, why some nebulae become bipolar during their lifetime is not well known. Various theories exist attempting explain it, but none are universally accepted as of yet. The model described here can be used to generate simulated nebulae which look similar to real observed bipolar nebulae. A qualitative comparison can give us a hint as to the density distributions of the real nebulae. Knowing the density distributions of real nebulae could help us understand the physical mechanisms that create the symmetry of bipolar nebulae. In addition to helping to discover the density distributions of bipolar nebulae, the program can be used to view results of theoretical simulations for bipolar formation so they can be compared with observations.

The model assumes steady, constant-velocity mass loss, which leads to a radial density distribution at any given latitude, θ, of $1/r^2$. More specifically,

$$\rho(\theta, r) = \frac{\dot{M}}{4\pi r^2 v} f(\theta) \tag{1}$$

where ρ is the density of both the dust and the gas, $\dot{M}$ is the total mass loss rate for the star, and v is the velocity with which the mass is ejected, assumed to be constant in this study. Because dust does not condense into grains at high temperatures, dust grains cannot exist inside a certain inner radius from the star, r_i. Inside that radius, the dust density should be zero. However, dust grain formation is a continuous process, so grains continue to form and grow over a finite radial distance. Therefore, the inner cutoff radius for dust is not a sharp discontinuity but a smoothed edge. To accommodate that factor, the model assumes a $1/r^2$ dust distribution, cut off at an inner radius r_i, and smoothed by convolution with a triangular function. This steady, constant-velocity radial distribution is the default for the model, however, alternative radial distributions can be chosen to use in a simulation by varying $\dot{M}(t)$ and $v(t)$. The latitudinal density distribution $f(\theta)$is specified by the user, and can be any function of latitude. In addition, the user specifies an angle of inclination, $\angle_{inc}$, at which to view the nebula.

To describe light as it moves through dust, the model utilizes the equation of radiative transfer and assumes single scattering. The equation of radiative transfer is [1]:

$$\frac{dI}{dx} = -\kappa I + \epsilon \tag{2}$$

where I is the intensity of a beam of light, x is distance in the direction of propagation, κ is a quantity called the absorption coefficient, and ϵ is the emissivity. The absorption coefficient represents the fraction of the intensity that is absorbed by the material, or scattered out of the beam. The emissivity represents any intensity that is emitted by the material into the beam, or that is scattered into the beam. The model integrates equation (2) twice; initially it integrates the intensity from the central star radially outwards to obtain values of radial intensity at a finite number of angles and radii, then it integrates the intensity along the line of sight of the viewer. When the intensity is integrated along the line of sight, the emissivity is represented by the fraction of the radial intensity at a given point (known from the first integration) that is scattered in the direction of the viewer. The absorption coefficient and emissivity are defined based on the cross sectional area of the scattering dust grains. The grains are assumed to be spherical, with a constant grain size. We assumed an isotropic scattering phase function.

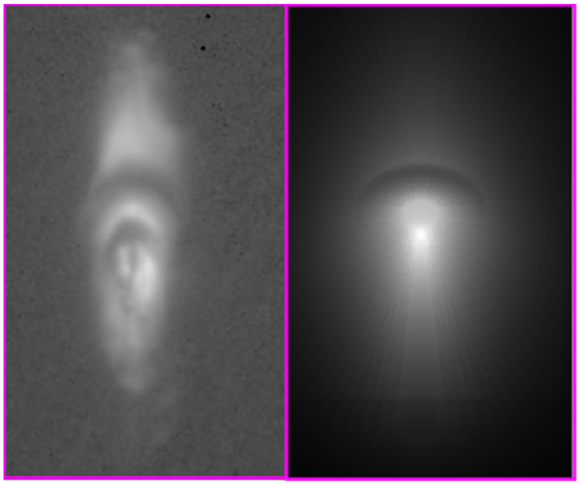

FIG. 1: Left: Real image of i22036, Right: Image from a simulation, $\dot{M} = 10^{-4} M_{sun}/yr$, $r_i = 3.3 \times 10^{14} cm$, $v = 10 km/s$, $\angle_{inc} = \pi/4 rad$

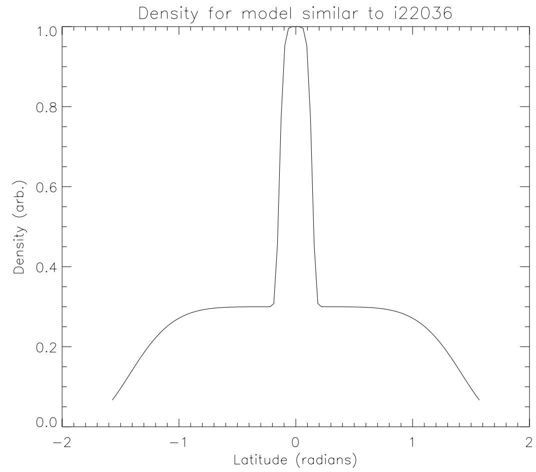

FIG. 2: The latitudinal density distribution of the model corresponding to IRAS22036+5306

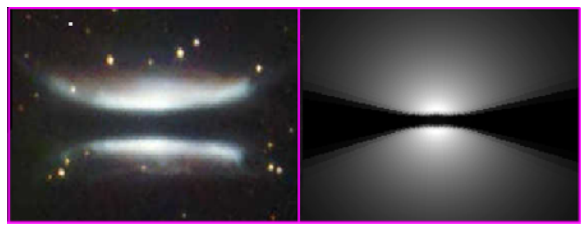

FIG. 3: Left: Real image of Gomez's Hamburger, Right: Image from a simulation, $\dot{M} = 10^{-4} M_{sun}/yr$, $r_i = 3.3 \times 10^{14} cm$, $v = 10 km/s$, $\angle_{inc} = 0 rad$

The absorption coefficient is defined as

$$\kappa = n\pi a^2 \frac{1}{4\pi} \tag{3}$$

where n is the number density of dust grains and a is the radius of each grain. The grain size is assumed to be constant. The emissivity is defined to be

$$\epsilon = n\pi a^2 \frac{1}{4\pi} I_r A \tag{4}$$

Here, I_r is the radial intensity and A is the albedo of the dust.

The integration is performed with a second-order Runge-Kutta scheme. In the radial integration, a variable step size is used. The steps are sized so that all steps have equal opacity when the radial density distribution goes as $1/r^2$. In the integration along the line of sight, a constant step size is used. For the radial integration, in addition to integrating equation (2), it is necessary to reduce the intensity by $1/r^2$ because the light spreads out as it moves radially. To do this, after each step in the radial integration, the intensity at a step k is redefined as follows:

$$I_k = I_k \frac{r_{k-1}^2}{r_k^2} \tag{5}$$

We created two model nebulae which are comparable to the real nebulae IRAS22036+5306 and Gomez's Hamburger. Both models were created using the default radial density distribution. They used 150 integration steps in the radial and line-of sight integrations, at 50 different latitudes. The central star was assumed to have a radius of $1AU$. The grain radius was $10^{-5} cm$, the bulk density of dust grains was $2.5 g/cm^3$, and the albedo was 0.25. The ratio by mass of gas to dust was 200.

In Figure 1, an image of the real nebula IRAS22036+5306 and one of the model nebula are shown next to each other. Qualitatively, the images have similarities, such as the dark equatorial disk which blocks light, and the egg-shaped bright area below the disk. There are also differences; for example the real nebula has very defined light areas along the axis of symmetry that the model does not have. This could be due to effects that our model does not take into account, such as jets along the axis of symmetry. In addition, there are asymmetric dark areas near the center of IRAS22036+5306 which obviously cannot be reproduced with our symmetric model. The latitudinal density distribution used in the simulation is shown in Figure 2, and consists of a very dense equatorial disk with a less dense background. The model nebula is viewed at an inclination angle of $\pi/4$ radians.

In Figure 3, an image of the real nebula Gomez's Hamburger and one of a comparative model nebula are shown. Again, the images have qualitative similarities. There is a large, dark disk, seen edge-on, and the only light that escapes does so on the top or bottom of that disk. Again, there are also differences, for example the real nebula has two fairly flat disks of light, and the simulated nebula

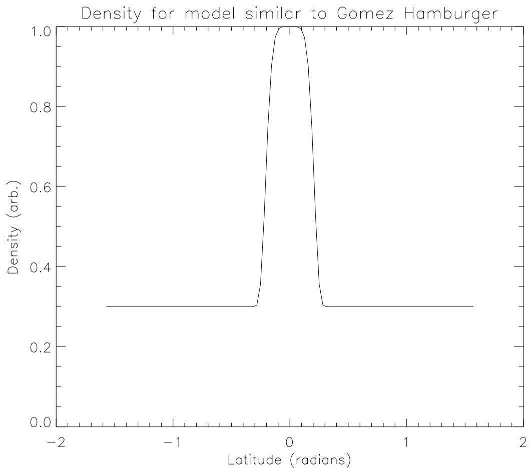

FIG. 4: The latitudinal density distribution of the model corresponding to Gomez's Hamburger

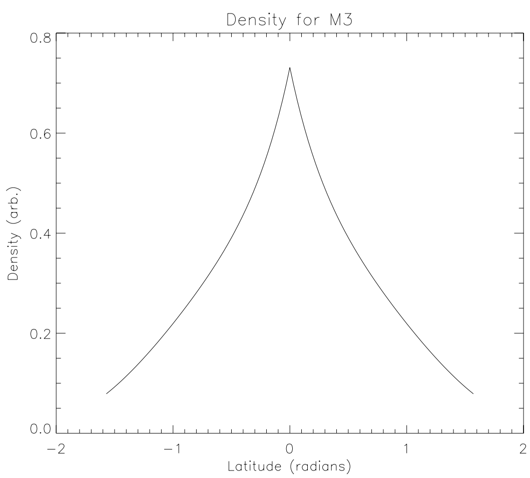

FIG. 5: A functional fit to the latitudinal density distribution produced in model M3 of Mastrodemos and Morris, 1999 [2].

has more rounded semi-circles of light. The latitudinal density distribution for the simulated nebula is shown in Figure 4, and again consists of a very dense equatorial disk plus a less dense background. The model is viewed at an inclination angle of 0 radians.

The effect of the systemic inclination can be seen by creating simulations of nebulae with identical parameters, but with different inclinations. To show this, we created models of the simulation seen in Figure 1 at varying inclinations. The results are shown in Figure 7, with inclinations of 0.1468 , $\pi/4$, and $\pi/2$ radians. The reason 0.1486 radians was selected as an angle of inclination is because it corresponds to the angle at the edge of the thick equatorial disk (see Figure 2).

We also used the model to view the results of simulations of physical processes theorized to cause bipolar nebulae. Specifically, we modeled results given by Mastrodemos and Morris, 1999 [2] for the dust density distribution resulting when a nebula's central star is part of a binary system. Mastrodemos and Morris created fifteen models, with different values for the secondary mass, semimajor axis, dust outflow velocity, and other parameters. Of those fifteen models, five were given a functional fit for their resulting latitudinal density distribution. We created images of all five of those models, and will show one here. The model we are showing is M3 from [2]. The functional fit for the latitudinal density distribution of M3 is given by Figure 5, and an image is given in Figure 6. We used the same values for star radius, grain radius, bulk density, albedo, and gas-to-dust ratio as in the previous two simulations.

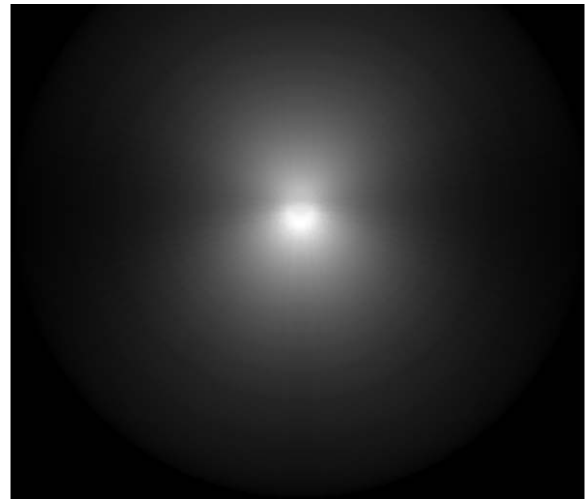

FIG. 6: An image of M3, $\dot{M} = 7.5 \times 10^{-5} M_{sun}/yr$, $r_i = 5.0 \times 10^{14} cm$, $v = 15km/s$, $\angle_{inc} = \pi/8 rad$

From the simulations shown here, we can draw some conclusions about the bipolar nebulae IRAS22036+5306 and Gomez's Hamburger. We were unable to create models resembling these nebulae without a sharp contrast between the high-density equatorial disk and the lower density areas at higher latitudes. In both cases, the dark disk around the waist of the nebula could not be reproduced without this sharp contrast. The image of the model M3, which did not have a well-defined equatorial disk, did not have the same kind of dark equatorial features seen in IRAS22036+5306 or Gomez's Hamburger.

We can also draw conclusions about the importance of inclination angle in viewing bipolar nebulae from our simulations of an identical nebulae at three different angles of inclination. The simulated nebula looks very different when viewed at different angles. As we would expect, the nebula viewed at an inclination angle of $\pi/2$ radians does not even look bipolar. A viewer looking at such an object from Earth would have no way of knowing it was not spherically symmetric. Similarly, a viewer looking at the nebula at an angle of 0.1468 radians might never think

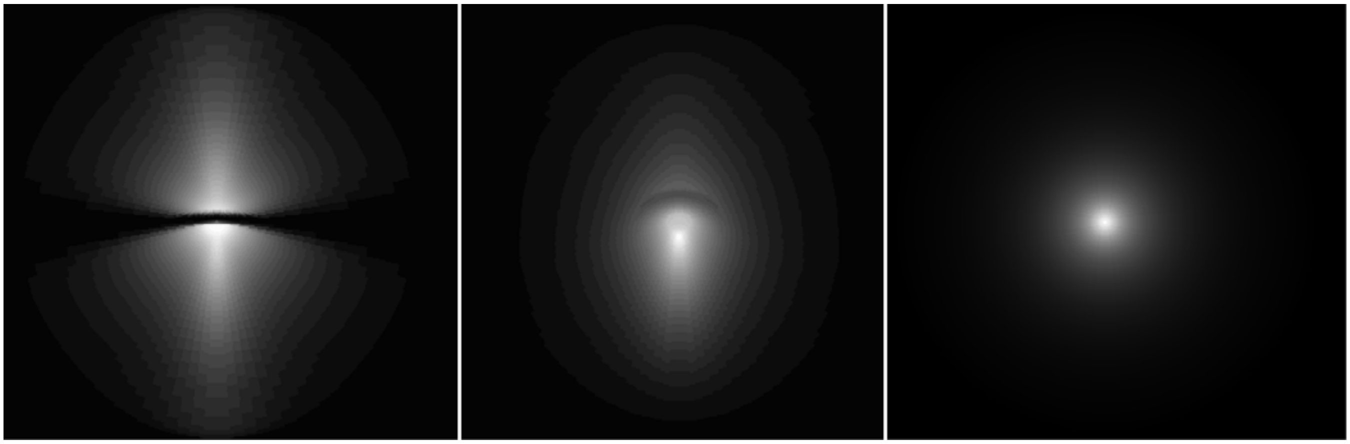

FIG. 7: Left: Image from a simulation, $\angle_{inc} = 0.1468$ rad, Middle: Image from a simulation, $\angle_{inc} = \pi/4$ rad, Right: Image from a simulation, $\angle_{inc} = \pi/2$ rad. All the parameters for the models are the same as in the simulated image shown in Figure 1, except for systemic inclination.

that the same object, viewed at an angle of $\pi/4$ radians, would look the way it does in Figure 7.

Our model can be used to guess the density distributions of observed bipolar nebulae and to check the results of models of physical processes. One of its greatest limitations, however, is that there is no way yet to quantitatively compare simulated images to real ones. Additionally, the model assumes a constant grain size, when in reality there would probably be a dependence of average grain size on radius. It also does not incorporate a scattering dependence on wavelength or a scattering phase function.

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[1] J. T. Jeffries, *Spectral Line Formation* (Blaisdell Publishing Company, 1968).

[2] N. Mastrodemos and M. Morris, The Astrophysical Journal **523**, 357 (1999).